Durable Goods and the Wealth Distribution*

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Abstract

This paper studies the role of durable consumption goods and collateral lending in shaping the wealth distribution in an otherwise standard heterogeneous agents model economy with idiosyncratic uncertainty. The model accommodates two kinds of assets, durable goods and financial assets, and therefore can be used to analyze wealth composition issues. We demonstrate that the model can reproduce several features found in the data: (i) The distribution of total wealth is much more concentrated than that of earnings, (ii) the distribution of financial assets is even more concentrated than that of total wealth, (iii) the distributions of earnings and durables tend to be very similar and (iv) durables as a fraction of total wealth decrease with wealth.

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1 Introduction

U.S. data shows that the composition of wealth (net worth) differs significantly across households. For instance, the 1998 *Survey of Consumer Finances* reports that the poorest 80 percent of households hold, on average, 129 percent of their wealth as real estate and automobiles, whereas this percent goes down to 29 for the richest 20 percent of the population. Moreover, the distribution of financial assets (net worth minus real estate and automobiles) is extremely unequal: the top 20 percent of households holds 101 percent of these assets. Moreover, 92 percent of all available credit to households is for the purchase of houses and automobiles (collateral credit). The average ratio of collateral credit to total debt across households is 78.7 percent. These figures suggest that for a high proportion of the population, net worth consists of durable goods minus debts, typically collateralized debts. These facts lead us to believe that the debt and wealth positions of households depend not only on their earnings history and the consumption stream that they choose, but on the financial conditions that determine the amount of collateral needed to obtain credit for the purchase of houses and automobiles. In this paper, we investigate the effects that the explicit consideration of the existence of durable goods and collateral credit have on the shape of the wealth distribution, wealth composition, and its effects on the level of precautionary savings. Our model is an otherwise standard heterogeneous agents model economy with idiosyncratic uncertainty.

We begin by documenting some features of the wealth distribution of the U.S. economy using the Survey of Consumer Finances. We report inequality statistics for three measures of household wealth–total wealth, durables, and financial assets–and for household earnings. *Total wealth* is defined as net worth: total assets minus total liabilities. The category *durables* is comprised of residential stock and automobiles, while *financial assets* are equal to net worth minus the value of the stock of durables. Thus, all debts, including mortgages, home equity loans and car loans, represent a negative position of financial assets. The first pattern that we encounter is that the distribution of total wealth is much more concentrated than that of earnings, their respective Gini indices being 0.796 and 0.611. The distribution of financial assets is even more concentrated than that of total wealth (its Gini index is 0.953). Durables and earnings have very similar distributions: their Gini indices are 0.626 and 0.611 respectively, while the mean to median ratio is 1.52 for durables and 1.57 for earnings. The similarity of these distributions is a feature that appears not only in the most recent (1998) Survey of Consumer Finances, but also in two previous surveys (1992 and 1989). Thus, we believe that this is an empirical regularity of the data. Finally, the value of durables as a fraction of total wealth decreases with the level of wealth: the bottom 40 percent of households hold 317 percent of their wealth as durables, whereas the top 20 percent only hold 29 percent of their wealth as houses and automobiles. Table 4 summarizes these facts.

In order to construct a model that accounts for all of these patterns, we need to take into account the specific features that differentiate durables from other assets. First, households derive utility from the services they provide but typically they are not part of productive capital (i.e., it is financial assets that usually materialize into productive capital). Second, there may be down payment requirements when buying durable goods. Third, durable goods can be used as collateral for credit. That is, individuals can borrow up to some fraction of the amount of durable goods that they own. Fourth, selling or buying durable goods may be subject to adjustment costs. People do not move every day or change the car they drive very often, and when they do, they encounter several costs (i.e., search costs, taxes and transaction costs).

We build a model economy of *ex ante* identical households who face uninsurable idiosyncratic shocks to their labor endowments that accommodates the four features of durables aforementioned. In our model, households save not only for precautionary reasons but to accumulate enough assets to satisfy the down payment required in the purchase of durable goods. Moreover, in this economy, households can borrow against the value of their holdings of durables. Thus, there are two reasons why households hold them: they provide services and they provide collateral for credit (i.e., durable equity loans). We consider the model with and without adjustment costs. For simplicity, we allow no other form of credit. We also assume that there is no rental market for durable services. We calibrate our model economy so its steady state statistics match some of the aggregate statistics of the U.S. economy and the data on earnings distribution. The wealth distribution and the wealth composition statistics delivered by the model resemble quite closely those of the data in all the dimensions previously discussed.

As in the data, durables have a distribution very similar to that of earnings. We find that this similarity depends on the persistence of household earnings. Expenditures on durables, as on any other consumption good, depend on the level of permanent income. If earnings are very persistent, permanent income is very correlated with current earnings and so are durables. In this case, durables and earnings have very similar distributions. If, on the contrary, earnings are very volatile, the correlation of permanent income and current earnings will be low and the durables distribution no longer resembles that of earnings.

Next, we consider the aggregate and distributional effects of changes in collateral (i.e., down payment) requirements. As down payments decrease, the ratio of financial assets to output decreases, the ratio of durables to output increases and total output decreases. As for the distributional effects, total wealth inequality increases and so does inequality in financial assets. The distribution of durables remains almost unchanged. The explanation is twofold. On one hand, a lower down payment implies that households can finance a higher fraction of the durables that they purchase. This makes inequality in financial assets and, hence, total wealth to increase. On the other hand, the amount of credit that they can obtain as durable equity loans is also higher. Thus, households fear less bad shocks in earnings and consequently the amount of their wealth held as durables, the illiquid asset, increases.

This paper contributes to the literature of general equilibrium model economies with a large number of *ex ante* identical agents with standard preferences subject to uninsurable, idiosyncratic shocks. In particular, we study the size of precautionary savings and the factors that account for the very large differences in asset holdings across households. The first paper to address this issue within an infinite horizon framework was Aiyagari (1994). Huggett (1996) uses a life-cycle framework to address the same issue. Quadrini and Ríos-Rull (1997) review the literature up to that date. Krusell and Smith (1998) defend shocks to preferences to account for wealth inequality while Carroll (2000) argues that we should use models where consumers consider the accumulation of wealth as an end in itself or models where wealth yields a large unobservable flow of services. More recently, Castañeda, Díaz-Giménez, and Ríos-Rull (2001) have provided a theory to jointly understand the distribution of earnings and net worth. They also provide an excellent review of the literature.

The explicit consideration of durable goods has proven to be useful for understanding the "excess smoothness" and "excess sensitivity" of consumption.[1] Farr and Luengo-Prado (2001) build a small open economy populated by *ex ante* identical households who receive uninsured idiosyncratic labor shocks and derive utility from durables services as well as nondurables, and where a down payment is required to purchase durables. The aggregate consumption path implied by their model displays a degree of smoothness and sensitivity comparable to that found in the data. We build upon Farr and Luengo-Prado's (2001) economic framework and construct a general equilibrium model to study the wealth distribution implications of the existence of durable goods. Fernández-Villaverde and Krueger (2001) also consider durables explicitly in a model of heterogeneous agents. Their focus is, however, the hump-shaped evolution of consumption expenditures over the life-cycle. Other authors have explicitly modelled durables to study the welfare effects of alternative monetary policies (see, for example, J. Díaz-Giménez and E. C. Prescott and T. Fitzgerald and F. Álvarez 1992), or to quantify the welfare consequences of imposing different levels of borrowing constraints (see, for instance, Díaz-Giménez and Puch 1998). In a representative agent setting, similar models have been used in some empirical applications (see, for instance, Chah, Ramey, and Starr 1995, Alessie, Devereux, and Weber 1997, Brugiavini and Weber 1994).

The remainder of this paper is organized as follows. Section 2 describes the economic environment and defines the equilibrium concept. Section 3 describes the calibration of our model economy and discusses the relationship of this economy with one that abstracts from the existence of durable goods. Section 4 presents the main results concerning the volume of precautionary savings and the wealth distribution for our benchmark model. In section 5, we study in detail the factors that account for the similarity of the durables and earnings distributions. We analyze the aggregate and distributional effects of changes in the down payment requirement in section 6. Brief concluding remarks are provided in Section 7.

2 The Model Economy

We consider a production economy populated by a continuum of households of measure one that live forever. We focus our analysis on steady states. Sections 2.1, 2.2 and 2.3 describe the technology, preferences and endowments, and the market arrangements. Section 2.4 presents the household problem and section 2.5 provides a formal definition of steady state equilibrium.

[1] Excess sensitivity refers to the correlation of aggregate consumption growth and past income growth. (see, for instance, Flavin 1989). Excess smoothness refers to the small response of aggregate consumption to innovations in permanent income. (see, for instance, Campbell and Deaton 1989).

2.1 Technology

Aggregate output, Y, is produced according to an aggregate neoclassical production function that takes as inputs aggregate capital, K, and aggregate labor, L: $Y = F(K, L)$.

The final good, can be either consumed, invested in capital or invested in durable goods on a one-to-one basis. Therefore, we can write the feasibility constraint as:

$$C + I_k + I_d = F(K, L), \tag{1}$$

where C is nondurable consumption, I_k is investment in capital and I_d is investment in durable goods. We assume that durable goods and capital depreciate at the rates δ^d and δ^k respectively.

2.2 Preferences and endowments

Households derive utility from the consumption of a nondurable good, c, and from the service flow, $s(d)$, provided by a durable good, d. They do not derive utility from leisure. We write the per period utility as $u(c, s(d))$ and lifetime utility as $\sum_{t=0}^{\infty} \beta^t u(c_t, s(d_t))$, where β is the time discount factor. For the purpose of this paper, durable goods are private residential assets plus automobiles.

Each period households receive a shock to their efficiency units of labor $e \in E = \{e_1, ..., e_{n_e}\}$. This shock is Markov with transition matrix $\pi_{e,e'}$.

2.3 Market arrangements

We assume that there are no state contingent markets for the household specific shock. Households hold durable goods $d \in [0, \infty)$ and financial assets $k \in [\underline{k}, \infty)$, $\underline{k} \in \mathbb{R}$. There is no rental market for durables. Financial assets pay a net interest rate r. For simplicity, we assume that there are no differences between borrowing and lending rates. Durable goods provide collateral for loans. In particular:

$$k \geq -(1 - \theta)d. \tag{2}$$

This liquidity constraint implies that the maximum debt an individual can incur is a fraction $(1 - \theta)$ of his durable stock, which determines the lower bound for k, $\underline{k}$. The constraint summarizes several aspects of collateral lending that we see in reality. First, it means that when purchasing a durable good, a household can only finance a fraction $(1-\theta)$ of it. In other words, it must satisfy a *down payment* requirement θ. The constraint also implies that when a household owns a durable good, it can obtain a loan for up to a fraction $(1 - \theta)$ of its value (*durable equity loans*). In summary, at any point in time, an agent is only required to keep an accumulated durable equity of θd. Note that the constraint also implies that the household cannot borrow if it does not own any durable good. In practice, financial institutions require down payments for a number of reasons. For example, down payments reduce the moral hazard problem in the care that owners take in maintaining the value of the durable and they also mitigate the effects of the adverse selection problem that results from asymmetric information in the credit market.

Finally, we assume that durable goods transactions are costly. In particular, we assume a non-convex cost of adjustment, $\tau(d', d)$.

2.4 The household's problem

The household's state variables are its labor endowment shock, its holdings of capital, and its holdings of durable goods, $\{e, k, d\}$. The problem that a household solves is:

$$v(e,k,d) = \max_{c \geq 0, d' \geq 0} u(c, s(d)) \; + \; \beta \sum_{e'} \pi_{e,e'} \; v\left(e', k', d'\right) \tag{3}$$

$$\text{s.t.} \qquad c + k' + d' - \left(1 - \delta^d\right) d + \tau(d', d) = w \; e + (1 + r) \; k, \tag{4}$$

$$k' + (1 - \theta)\, d' \geq 0. \tag{5}$$

It is well known that under certain conditions, problems of this type have a solution that we denote $k' = g^k(e,k,d)$, $d' = g^d(e,k,d)$, $c = g^c(e,k,d)$ with an upper and a lower bound on capital holdings, $\{\underline{k}, \overline{k}\}$, and an upper bound on the stock of durable good, $\overline{d}$, such that $\overline{k} \geq g^k(e,k,d) \geq \underline{k}$ and $\overline{d} > g^d(e,k,d) > 0$, for all $e \in E$, all $d \in \left\{d \mid 0 \leq d \leq \overline{d}\right\}$, and all $k \in \left\{k \mid \underline{k} \leq k \leq \overline{k}\right\}$. For notational efficiency, we denote $x = \{e, k, d\}$ and $X = \{E \times [\underline{k}, \overline{k}] \times [0, \overline{d}]\}$. With respect to capital, the required condition is that we have a low enough rate of return, $\beta < \frac{1}{1+r}$ (see Aiyagari (1994), Huggett (1993) or Quadrini and Ríos-Rull (1997) for details).

It is possible to construct a Markov process for the individual state variables, from the Markov process on the shocks and the decision rules of the agents (see Huggett (1993) or Hopenhayn and Prescott (1992) for details). Let $\mathcal{B}$ be the σ-algebra generated in X by, say, the open intervals. A probability measure μ over $\mathcal{B}$ exhaustively describes the economy by stating how many households are of each type. Note that the first moment of μ over e yields the aggregate labor input while the first moment over k yields aggregate capital.

Let $P(x, B)$ denote the probability that a type $\{x\}$ has of becoming of a type in $B \subset \mathcal{B}$. The function P naturally describes how the economy moves over time by generating a probability measure for tomorrow μ' given a probability measure, μ, today. The exact way in which this occurs is

$$\mu'(B) = \int_X P(x, B) \; d\mu. \tag{6}$$

If the process for the earnings shock is normal in the sense that it has a unique stationary distribution, then the economy will also have a unique stationary distribution.[2]

[2]For example if it satisfies the *"American–dream American–nightmare"* condition stated in Ríos-Rull (1995), then there is a unique stationary distribution of households over earning shocks, assets holdings and stock of habits.

Furthermore, this unique stationary distribution is the limit to which the economy converges under any initial distribution.[3]

2.5 Equilibrium

We have almost all the ingredients to define a steady state equilibrium. We only need to add the condition that marginal productivities yield factor prices as functions of μ. Note that to obtain a steady state, we look for a measure of households μ such that given the prices implied by that measure, households actions reproduce the same measure μ in the following period. Formally, a steady state equilibrium for this economy is a set of functions for the household problem $\{v(x), g^k(x), g^d(x), g^c(x)\}$, and a measure of households, μ, such that:

1. Factor inputs are obtained aggregating over households: $K = \int_X \ k \ d\mu$, and $L = \int_X \ e \ d\mu$.
2. Factor rental prices are factor marginal productivities, $r = F_1(K, L) - \delta^k$, and $w = F_2(K, L)$.
3. Given μ, K, L and D, the functions $\left\{v(x), g^k(x), g^d(x), g^c(x)\right\}$ solve the households' decision problem described in Subsection 2.4.
4. The markets for capital, nondurables, and durables clear: $\int_X \left[g^k(x) - \left(1 - \delta^k\right) k\right] d\mu = I_k$, $\int_X g^c(x) \ d\mu = C$, $\int_X \left[g^d(x) - \left(1 - \delta^d\right) d + \tau \left(g^d(x), d\right)\right] d\mu = I_d$.
5. The measure of households is stationary: $x(B) = \int_X \ P(x, B) \ d\mu$, for all $B \subset \mathcal{B}$.

3 Calibration

In this section we discuss the calibration of our model economy. We are going to compare the predictions of the model with those delivered by a standard one-asset economy that abstracts from the composition of wealth. Before doing so, we discuss the relationship between both frameworks as this will help us to understand their different implications on wealth distribution.

3.1 The model versus a one-asset economy

Our goal is to explore the role of durable goods in quantitatively shaping the wealth distribution. In order to do this, we must compare our model economy, which we call a *two-asset economy*, with a *one-asset economy*, essentially a version of Aiyagari (1994). The comparison of both economies is not straightforward.

Assume that there is a perfect rental market for durable goods. The rental market for durable services operates in the following way: all households pool their stock of durables together, receive a rent from the flow of services of the stock they own, $r^s s(d)$, and pay

[3]This does not mean that this will happen in equilibrium outside the steady state since the transition P has been constructed under the assumption that households think that prices are constant.

the value of the services they consume, $r^s s$. Ignoring adjustment costs, the household's budget constraint would be:

$$c + r^s s + k' + d' - \left(1 - \delta^d\right) d = we + (1 + r)\, k + r^s s(d).$$

Note that in the one-asset economy, no down payment is required when buying a new unit of the durable good. In equilibrium, the rental price of capital and durable goods should be the same. Therefore, the composition of the household's portfolio is irrelevant when studying the distribution of wealth and we could think of capital and durables as a single asset.

The two-asset economy differs from a one-asset economy in two important aspects. First, consumption of nondurables and durable services are not perfect substitutes and, second, the stock of durables is not part of the stock of productive capital. Regarding household preferences, we cannot assume that both types of goods are perfect substitutes, otherwise the demand for each good would either be zero or indeterminate, depending on their relative prices. With regards to technology, private residential assets are part of the stock of productive capital in a one-asset economy whereas in our model they are not, and therefore, both economies should differ in their share of capital in the aggregate production function. We know that the equilibrium interest rate of our economy depends on the capital share in the production of the final good and on the output-capital ratio. Given that the durable stock is not part of the stock of capital used in the production of the final good, the share of capital should be smaller in the two-asset economy.

In summary, the comparison that we make in Section 4 could be thought of as comparing an economy without a rental market for durables, with the collateralized credit constraint and adjustment costs, with an economy with a perfect rental market and without the constraint or adjustment costs, taking into account that comparing both economies requires a detailed specification of what we call output and aggregate capital in each model. The calibration procedure and the data used are described in detail in Appendix A.

3.2 The earnings process

With respect to the process for earnings, Aiyagari (1994) sets an AR(1) in the logarithm of labor income. The process is fully described by two parameters: its persistence and its volatility. He chooses both values following estimates of Kydland (1984) that used PSID data and of Abowd and Card (1987) and Abowd and Card (1989) that used both PSID and NLS data. Then, he approximates the process by using a seven-state Markov chain following the procedures described in Tauchen (1986). Importantly, Aiyagari (1994) does not succeed in accounting for the degree of wealth inequality in the U.S. The main reason for this failure is that the earnings process that he uses is much more egalitarian that what we see in the data. In his benchmark calibration, the Gini index for earnings is 0.10, while it is around 0.60 for the U.S. economy. We use an earnings process very similar to the one used by Díaz, Pijoan-Mas, and Ríos-Rull (2000), which is constructed to match

Table 1: THE EARNINGS PROCESS

$$e \in \{e_1, e_2, e_3\} = \{1.00, 5.29, 46.55\}$$

$$\pi_{e,e'} = \begin{bmatrix} 0.96500 & 0.00347 & 0.000333 \\ 0.03937 & 0.95000 & 0.010625 \\ 0.00000 & 0.08300 & 0.917000 \end{bmatrix}$$

Stationary distribution			
$\pi^* =$	0.4983	0.4429	0.05870

the Lorenz curves of the U.S distributions for earnings and total wealth.[4] In Section 5, we show some results for a process with lower persistence.

3.3 Technology, preferences and market arrangements

For preferences over consumption of the nondurable good and services from the durable good we choose $u(c_t, s(d_t)) = \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \gamma \frac{(d_t+\varepsilon)^{1-\sigma}}{1-\sigma}\right)$ as in Farr and Luengo-Prado (2001).[5]

We assume that durable services are proportional to the durable stock and we choose the constant of proportionality to be one. This assumption is not restrictive, since we calibrate γ to match the ratio of nondurable goods to investment in durable goods, $C/I_D = 5.762$, that we see in the data. We set the discount factor β so that net interest rate in the steady is 4.633%. We choose $\sigma = 3$.[6]

Feasibility in our model is given by expression (1). Thus, in this model aggregate output corresponds to measured GDP minus housing services. The share of capital is 0.18639. We set depreciation rates so that $\delta^k = \frac{I^k/Y}{K/Y} = \frac{0.1189}{1.4673} = 0.081$ and $\delta^d = \frac{I^d/Y}{D/Y} = \frac{0.1310}{1.4384} = 0.09128$.

The parameter θ of the borrowing constraint is set equal to 0.3 to roughly match the average down payment for cars and houses during the period that we consider, 1954-1999. Thus, individuals can borrow up to 70 percent of the value of their holdings of durable good. The procedure for obtaining these numbers is detailed in Appendix A and parameter values are summarized in Table 2.

We consider non convex costs of adjustment as proposed by Grossman and Laroque (1990), which generate (S, s) adjustment behavior. In most periods, consumers do not adjust their durable stock, but when they do, they usually make substantial changes. The specification is:

$$\tau(d', d) = I\rho(1 - \delta^d)\, d, \tag{7}$$

[4]See Castañeda, Díaz-Giménez, and Ríos-Rull (2001), and Díaz, Pijoan-Mas, and Ríos-Rull (2000) for a discussion on this calibration choice.

[5]We set $\varepsilon = 0.01$

[6]Results did not change qualitatively for $\sigma = 1$ and $\sigma = 2$ and nonseparable utility functions.

where:

$$I = \begin{cases} 0 & \text{if } |d' - (1-\delta^d)\, d| \leq \varphi\, d, \\ 1 & \text{otherwise.} \end{cases}$$

Note that this cost can be seen as a loss in the selling price when making substantial changes of the durable good. However, we also allow the agent to make small changes in his durable stock without paying adjustment costs. ρ is the parameter controlling the proportional loss in the selling price while φ determines what a small change may be as a proportion of the owned durable stock. In our benchmark case we set ρ equal to 5% and φ equal to 10%.[7]

Figure 1 illustrates the (S, s) behavior that the specification generates. The figure depicts the optimal choice of durables for an agent who receives the best earnings shock every period, with and without adjustment costs.

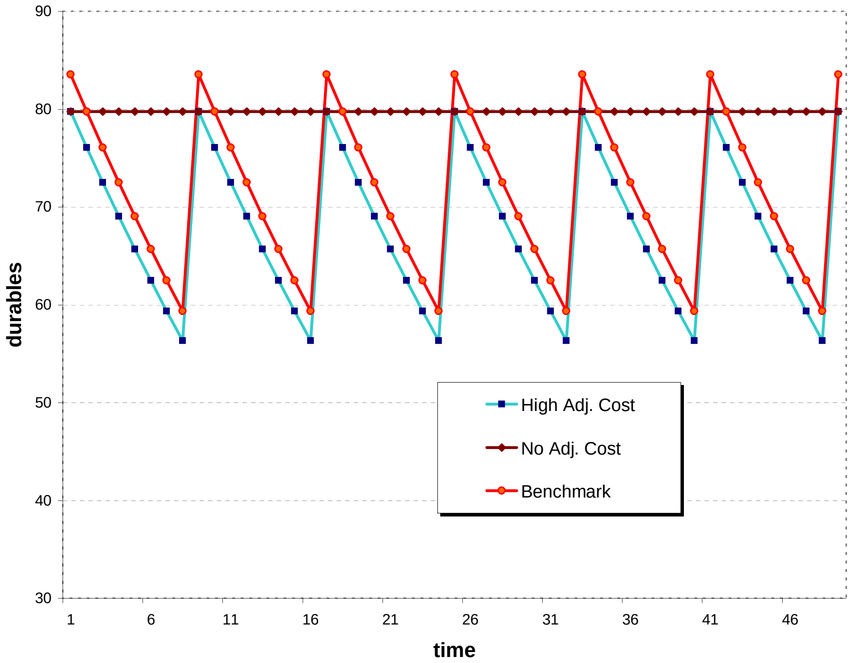

Figure 1: D with and without Adjustment Costs. $e = 46.55$

3.4 The one-asset economy

Preferences over consumption of the unique good are $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$. We choose $\sigma = 3$. The share of capital is 0.3480 for a capital-output ratio target of 2.51. We set the discount factor β so that net interest rate in the steady is 5.23 percent. We calibrate the depreciation rate so that the investment-capital ratio is equal to 0.0871 as in the data.

[7]Different specifications of the adjustment cost parameters did not change the qualitative results of our analysis.

Table 2: BASIC PARAMETERS FOR THE MODEL WITH TWO ASSETS

	Utility			Technology			Market Arrangements		
	β	σ	γ	α	δ_k	δ_d	θ	ρ	φ
Two Assets	0.758	3	1.5	0.186	0.081	0.091	0.3	0.05	0.1
One Asset	0.783	3	-	0.348	0.087	-	-		

The procedure for obtaining these figures is detailed in Appendix A and parameter values are summarized in Table 2.

4 Results

In this section we compare our benchmark two-asset economy and the economy with one-asset in two dimensions: the aggregates and their wealth distribution implications.

4.1 Aggregate savings

In order to analyze how the explicit consideration of durable goods changes the volume of aggregate savings, we compare each economy to its deterministic counterpart (the case with no uncertainty). Both economies are calibrated to produce comparable aggregates. Aggregate ratios are summarize in Table 3.

In the one-asset economy, the capital-output ratio more than doubles when idiosyncratic uncertainty is present (increasing from 0.96 to 2.51). Precautionary savings can be measured as the increase in the capital stock due to idiosyncratic uncertainty. This increase is 339 percent.[8]

Let us now turn to the two-asset economy. When there is uninsurable idiosyncratic risk, the durable-output ratio decreases slightly (compared to the case without uncertainty) while the capital-output ratio increases dramatically. The reason for these changes is that the existence of the down payment makes durables an illiquid asset. Consequently, households decrease their durable holdings and increase holdings of financial assets as a response to idiosyncratic uncertainty. Aggregate savings, $K + D$, increases by a 116 percent over the case without uncertainty. Note that aggregate savings can be decomposed as:

$$K + D = \theta D + K + (1 - \theta)D = \theta D + Q$$

That is, we can distinguish two types of savings: forced and voluntary. Forced savings are θD, the required down payment. Voluntary savings are given by *voluntary equity*, defined as $Q = K + (1 - \theta)D$, the equity held in excess of the required down payment. Thus, voluntary equity measures the volume of resources ready to smooth any bad shock,

[8] Our earnings process is very similar to Díaz, Pijoan-Mas, and Ríos-Rull (2000). However, these authors find that the increase in capital stock due to precautionary savings is only 134 percent. The reason for this discrepancy is in the calibration. They target a interest rate of 4.02 percent, and set the capital share equal to 0.36 and a risk aversion parameter equal to 2.

without changing the durable stock. The change in this stock due to uncertainty is 133 percent. Financial assets, K, increase by a 276 percent. Note that the effect of uncertainty on aggregate savings is smaller in the two-asset economy.

Table 3: AGGREGATE RATIOS

	Model		One Asset		*U.S. Data*	
	Determ.	Benchmark	Determ.	Benchmark	Two Assets	One Asset
Output	1.00	1.28	1.00	1.67	-	-
$\frac{K}{Y}+\frac{D}{Y}$	1.67	2.82	-	-	*2.91*	-
$\Delta(K+D)$	-	115.51%	-	-	-	-
$\frac{K}{Y}$	0.47	1.37	0.96	2.51	*1.47*	*2.51*
ΔK	-	276.05%	-	339.38%	-	-
$\frac{D}{Y}$	1.21	1.45	-	-	*1.44*	-
$\frac{K}{Y}+(1-\theta)\frac{D}{Y}$	1.31	2.38	-	-	*2.47*	-
$\Delta(K+(1-\theta)D)$	-	132.63%	-	-	-	-
$\frac{C}{I_D}$	7.77	5.76	-	-	*5.76*	-
Interest Rate	31.92%	4.63%	27.71 %	5.23%	*4.63%*	*5.23%*

4.2 Wealth distribution

Wealth distribution implications are summarized in Table 4. We present quintile shares, Gini coefficients and the mean to median ratio for some key variables–earnings, durables, financial assets and total wealth–in the benchmark model and in the data (*Survey of Consumer Finances*, 1998). For total wealth, we also report distribution measures for the model with one asset. Note that households are ranked according to the variable specified in each row. We also report wealth composition, defined as the fraction of durables in total wealth.

Table 4 shows that the distribution of earnings is slightly less concentrated than in the data. Calibrated earnings perform relatively worse for the bottom quintile, but it is important to keep in mind that in the model, households always have positive earnings by construction, while they can be negative in the data, due to losses reported by the self-employed.

The distribution of durables is very similar to that of earnings, as in the data. This similarity of both distributions is a feature that appears not only in the 1998 *Survey of Consumer Finances*, but also in the years 1992 and 1989. Thus, we believe that it is an empirical regularity. Our model accurately captures this resemblance. We discuss the reasons that allow our model to reproduce this feature of the data in Section 5.

Note that inequality in financial assets is almost as high as in the data and considerably higher than inequality in durables. Thus, durables are less concentrated than financial assets. We should not forget that durables are consumption goods, as well as assets, and therefore their distribution should probably be more equal than that of financial wealth. Moreover, since households can borrow to finance the purchase of durable goods, the level of inequality in financial assets will always be higher than in durables in the model (financial assets can be negative, while the durable stock has zero as its lower bound).

Total wealth is more unequal than earnings and durables are more equal than financial assets in the data and in the model. Note that total wealth inequality (or net worth inequality) in our model is slightly lower than in the standard one-asset economy. This is because the dispersion of durable goods is very small, which implies a lower index for net worth.

In summary, in the data, financial assets are more unequal than total wealth, total wealth is more unequal than earnings, and the distributions of durables and earnings are strikingly similar. Our model can reproduce all of these patterns reasonably well.

Let us now turn to the implications of our model for the composition of wealth. Table 4 shows the percentage of household net worth held as durables. We can see that this fraction decreases across quintiles, as in the data. The main explanation for this behavior is the concavity of the utility function. Part of the return to durables is the marginal utility from the consumption of its services, which is decreasing. As a result, durables as a fraction of total wealth must diminish, since the return to competing financial assets, the interest rate, is constant. The model fares very well in this dimension for the bottom 40 percent of households. This means that we are able to explain the level of debt of the poorest 40 percent of households. Its predictions for the other quintiles, however, are less satisfactory. Nevertheless, we should keep in mind that our model abstracts from differences in taxation for durables and financial assets, and more importantly, from life cycle effects in the durables market.

Table 4: Wealth Distribution and Wealth Composition

		Quintiles					
	Share (%)	1st	2nd	3rd	4th	5th	Gini
Earnings	Benchmark	3.53	3.53	11.62	18.7	62.62	0.602
	US Data	*-0.20*	*4.00*	*13.00*	*22.90*	*60.30*	*0.611*
Durables	Benchmark	3.40	3.93	12.21	16.97	63.49	0.599
	US data	*0.22*	*2.92*	*12.78*	*22.44*	*61.64*	*0.626*
Financial	Benchmark	-3.42	-2.52	-0.72	2.06	104.60	0.918
Assets	*US Data*	*-9.52*	*-1.07*	*0.36*	*5.84*	*104.39*	*0.953*
Total	Benchmark	0.52	0.77	6.34	9.22	83.15	0.777
Wealth	One Asset	0.00	0.51	4.89	8.02	86.58	0.801
	US data	*-0.30*	*1.35*	*5.14*	*12.37*	*81.44*	*0.796*
Wealth	Benchmark	316.96		113.47	94.02	46.69	
Composition	*US Data*	*317.08*		*120.28*	*78.12*	*29.2*	

5 The Distribution of Durables and Earnings

The question that we address in this section is why durables and earnings distributions are so similar. We hypothesize that the similarity in the distributions of earnings and durables depends on the persistence of household earnings. In order to verify this, we build a model economy identical in every respect to our benchmark economy except in the persistence of households earnings, that have a much higher volatility. We find that higher volatility leads to a much more egalitarian wealth distribution in all its three dimensions: total wealth, financial assets and durables. In this case, the distribution of durables no longer resembles that of earnings. The reason is that the distribution of durables depends on the distribution of permanent income. If the earnings process is very persistent, permanent income is highly correlated with that of earnings and so is the distribution of durables. If, on the contrary, earnings are very volatile, permanent income correlation with current earnings is low and so is the correlation of durables with earnings.

Permanent income and earnings

Since we abstract from aggregate uncertainty in this model economy, for any household whose state at period t is described by the 3-tuple $\{e_t, k_t, d_t\}$, we can write permanent income as the sum of current and future earnings plus the value of current financial assets and durable assets:

$$E_t \left\{ w \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} e_s \right\} + (1+r)k_t + \left(1 - \delta^d\right) d_t.$$

We can simplify the previous expression in the following way. Note that permanent earnings depend on the value of the current labor shock e_t and on the transition probability given by the Markov process π. Since we just focus on steady state equilibrium, we could compute a "permanent earnings shock" in the following way:

$$\widehat{e} = e + \frac{1}{1+r} \sum_{e'} \pi_{e,e'} \widehat{e}'. \tag{8}$$

Permanent income becomes $w\widehat{e} + (1+r)k + \left(1 - \delta^d\right) d$. Clearly, the higher the persistence in the labor endowment process e, the higher the correlation of current earnings and permanent income. If the liquidity constraint is not binding, consumption of non-durables and durables is governed by permanent resources. This should be the key to understanding the similarity of the distributions of earnings and durables: if the earnings process is very persistent, permanent income and earnings will be very correlated and so will be durables and earnings.

In order to verify this hypothesis, we have constructed a measure of permanent earnings using expression (8) for our model economy. The implied "permanent earnings shocks" are $\widehat{e} \in \{70.96, 134.91, 458.24\}$, which after being normalized by $\widehat{e}_1$, become $\widehat{e} \in \{1.00, 1.90, 6.46\}$. Thus, permanent earnings are not as unequal as current earnings.

Table 5: DURABLES, EARNINGS AND PERMANENT INCOME

	Quintiles Share (%)				
	1st	2nd	3rd	4th	5th
	A. Benchmark Model				
EARNINGS	3.53	3.53	11.62	18.70	62.61
PERMANENT INCOME	9.96	10.02	16.11	20.10	43.82
PERMANENT INCOME BY EARNINGS	10.43	10.44	16.49	21.62	41.03
DURABLES	2.67	3.53	12.12	17.57	64.11
DURABLES BY PERMANENT INCOME	2.67	3.57	12.87	17.32	63.57
	B. *Volatile* Earnings Model				
EARNINGS	3.64	3.64	11.27	19.23	62.23
PERMANENT INCOME	16.70	17.59	18.50	20.76	26.46
PERMANENT INCOME BY EARNINGS	18.83	18.73	19.27	19.83	23.28
DURABLES	7.68	11.56	11.84	25.62	38.30
DURABLES BY PERMANENT INCOME	8.32	12.01	16.33	25.11	38.22

All Parameters as in Table 2. The earnings process for benchmark model described in Table 1. The *Volatile* earnings process as in Table 6.

We then compute permanent income and calculate some basic correlations. The coefficient of correlation between durables and permanent income is 0.946, and the correlation between permanent income and current earnings is 0.971.

Table 5, panel A, shows the implied distribution of household permanent income with households ordered by permanent income and by current earnings. As we can see, both distributions are very similar. That is, the permanent income distribution depends primarily on earnings. We also report the distribution of household durables ranked by permanent income and by durables. Again, the distribution of durables is almost the same. This amounts to saying that permanent income is the main determinant of the distribution of durables: durables follow earnings because permanent income follows earnings.

An alternative experiment

We conduct a second experiment to further explore the hypothesis that durables are similar to earnings if permanent income is similar to earnings. We simulate our model economy using a different transition matrix for the earnings process, holding the earnings shocks and the stationary distribution for earnings constant. One way to do this is to assume that earning shocks tomorrow have constant probabilities independent of shocks today and equal to the stationary distribution probabilities. We call this process the *volatile* earnings process, which is presented in Table 6.

We compute permanent income and permanent earnings for the model economy. In this case, the implied "permanent earnings shocks" are $\widehat{e} \in \{125.03, 129.32, 170.58\}$, which after being normalized by $\widehat{e}_1$, become $\widehat{e} \in \{1.00, 1.03, 1.36\}$. Thus, permanent income in this case is much less concentrated than earnings. Table 7, shows the distributions of earnings, durables, financial assets and total wealth. The distribution of earnings is not affected by the change in the transition matrix since the stationary distribution remains the same but the distribution of wealth changes dramatically. As we can see, the Gini

Table 6: A VOLATILE EARNINGS PROCESS

$e \in \{e_1, e_2, e_3\} =$	$\{1.00, 5.29, 46.55\}$
$\pi_{e,e'} =$	$\begin{bmatrix} 0.4983 & 0.4429 & 0.0587 \\ 0.4983 & 0.4429 & 0.0587 \\ 0.4983 & 0.4429 & 0.0587 \end{bmatrix}$
Stationary distribution	
$\pi^* =$	0.4983 0.4429 0.0587

Table 7: WEALTH DISTRIBUTION AND COMPOSITION WITH VOLATILE EARNINGS

	Quintiles Share (%)					
	1st	2nd	3rd	4th	5th	Gini
DURABLES, D	7.81	11.55	16.72	26.00	37.91	0.315
FINANCIAL ASSETS, K	-3.45	0.31	6.47	28.99	67.68	0.724
TOTAL WEALTH, $K + D$	2.70	6.17	11.75	27.36	52.01	0.506
$D/(K + D)\%$	186.08	99.28	75.82	50.74	38.77	

index for total wealth is much lower now, 0.506, and that for durables is just 0.315. Thus, the first lesson that we learn from this exercise is that persistence is needed to match the wealth distribution in its three dimensions.

Panel B of table 5 shows the implied distribution of household permanent income ordered by permanent income and earnings. As we can see, both distributions are very similar. We also report the distribution of household durables ranked by permanent income and by durables. Again, quintiles are similar, but now the distribution is much more egalitarian than in the benchmark model. The reason is that permanent income distribution is much more egalitarian as well. The coefficient of correlation between durables and permanent income now is 0.919, and the correlation between permanent income and earnings is only 0.704.

What happens in the data?

We have explained what makes earnings and durables distributions so similar in our model. Is this a plausible explanation for this feature of the data? Table 8 presents the distribution of durables ordered by earnings in both model economies, the benchmark economy with persistent earnings and the economy with volatile earnings shocks. The table also reports the distribution of durables in the data, for households whose head is of age 30-65 so that we can abstract from life-cycle considerations.

As seen in the table, the model with persistent earnings approximates the data better than the model volatile earnings shocks. This may be seen as indirect evidence of persistency of earnings in reality. Since we are dealing with an infinite-horizon model, this can also be interpreted as a sign of relatively low social mobility.

Table 8: Durables and Earnings: Model and Data

	Quintiles Share (%)				
	1st	2nd	3rd	4th	5th
Benchmark Model	5.24	5.08	14.00	22.73	52.95
Volatile Earnings Model	17.85	17.91	19.11	19.14	25.99
U.S. Data	*5.69*	*10.14*	*14.57*	*20.93*	*48.64*

Note: Durables ordered by household earnings.

6 Changes in the Down Payment Requirement

In this section, we study the aggregate and distributional effects of changes in the down payment requirement. We focus on two issues with this experiment: first, the effects of changes in financial conditions on total wealth and wealth composition, and second, the volume of precautionary savings when no collateralized borrowing is allowed. We consider each issued in turn.

6.1 Changes in financial conditions

Over the last few decades we have seen a significant reduction in the down payment required by financial institutions for collateral lending as well as the proliferation of home equity loans. In this subsection, we study the effects of such measures on aggregate ratios and on the distribution of assets. In our model, a decrease in the parameter θ represents these financial improvements (although it cannot disentangle one from the other).

We simulate our model economy for different values of the down payment. Table 9 shows the main aggregate statistics and the Gini indices for different values of θ. Note that the capital-output ratio is lower the lower the down payment, whereas the durable-output ratio increases (until the very end); thus, aggregate output falls. For instance if the down payment falls from 50 percent to 30 percent, aggregate wealth falls by 1.51 percent and aggregate output falls by 1 percent. Moreover, the overall inequality of wealth increases as the down payment decreases. The reason is the following: households want to buy durable goods; if the down payment is lower, they can afford higher a consumption of durable goods at low levels of income but they need to borrow more. Durable good dispersion remains fairly constant, but financial wealth varies greatly, thus accounting for the increase in total wealth inequality. The changes in the distribution of financial assets are shown in Table 10. The composition of wealth for the bottom 40 percent of households varies significantly with changes in the down payment. The reason is, of course, that households for which the liquidity constraint is binding are concentrated in those two quintiles.

Table 9 also shows that total wealth decreases with a decreasing down payment until the very end. A lower down payment implies that a lower fraction of wealth has to be put aside as required durable equity. Moreover, a lower down payment means that a household can obtain a larger line of credit for the same value of collateral. This implies that households may not need to accumulate as many assets to insure themselves against bad earnings shocks. Note, however, that the decrease in total wealth is small. The reason is that voluntary equity, Q, increases. Since durable required equity is lower for lower down payments, households choose to increase voluntary equity and total wealth only decreases slightly.

Thus, a decrease in the down payment implies a small total wealth reduction, slightly lower aggregate output and higher inequality caused by a higher concentration of financial assets. Wealth composition is significantly affected, with the poor increasing dramatically their durable holdings as a proportion of total wealth. Note that because of the general equilibrium component of our model, the interest rate r increases as the down payment decreases.

What happens at the very end for $\theta = 0.2$? The behavior of the main aggregates and the various measures of inequality is reversed. The reason is the following. There are two effects when the down payment falls. The one just described in the previous paragraphs is a partial equilibrium effect that would be present if the interest rate where held constant. The other effect is an indirect, general equilibrium effect, induced by the increase in the interest rate. As the interest rate rises households increase their holdings of financial assets and decrease those of durables. This drives up the capital-output ratio and implies a lower inequality in financial assets and, hence, in total wealth. To illustrate the difference between both of them we report in the last column of Table 9 a partial equilibrium exercise for a down payment of 20 percent holding constant the interest rate. Note that in this case, the changes in the main aggregates and the various measures of inequality are in line with the effects previously described for higher levels of the down payment. In summary, an easing of financial conditions implies that durables become more liquid and hence more affordable and attractive. Nevertheless, the return to financial assets increases too. The effect of the increase in the return of durables is what we have called the partial equilibrium effect, whereas that of the increase in the return to financial assets is the general equilibrium effect. The latter dominates the former for a sufficiently low down payment.

6.2 Durability in consumption and precautionary savings

In this section we want to further investigate the volume of precautionary savings implied by the explicit consideration of durables and the existence of collateralized borrowing. Thus, we calculate the change in aggregate savings and voluntary equity due to the existence of idiosyncratic uncertainty in each model economy, characterized by a different down payment. Table 11 shows the level of output in each case.[9] The increase in total wealth is very similar in all cases, but the change in voluntary equity is larger the higher the down payment. This is reasonable since the larger the down payment, the lower is the possibility of borrowing in the economy to smooth out bad shocks. In the extreme case in which the down payment is equal to one, ($\theta = 1$), no borrowing is allowed. This economy differs from the one asset economy reviewed in section 4 only in the explicit existence of durables. Precautionary savings in the one-asset economy imply an increase of 339 percent increase in the aggregate capital stock. For the case in which ($\theta = 1$), precautionary savings imply a 321 percent increase in voluntary equity (in this case equal to the capital

[9] We have normalized all variables so that output in the certainty case is one for each model economy characterized for a different down payment.

Table 9: AGGREGATE RATIOS AND INEQUALITY INDEXES FOR DIFFERENT DOWN PAYMENTS

θ	1	0.5	0.3	0.2	0.2*
$(K+D)/Y$	2.87	2.84	2.82	2.87	2.81
K/Y	1.50	1.40	1.37	1.44	1.36
D/Y	1.37	1.44	1.45	1.43	1.45
Q/Y	1.50	2.12	2.38	2.59	2.53
C/I_D	6.04	5.77	5.76	5.80	5.71
r	3.73%	4.61%	4.63%	5.41%	4.63%
$\Delta(K+D)$	4.16%	1.51%	–	3.35%	–0.36%
ΔQ	-35.73%	-10.41%	–	10.04%	6.30%
Gini Index, K	0.864	0.912	0.919	0.907	0.922
Gini Index D	0.607	0.601	0.599	0.605	0.595
Gini Index, $K+D$	0.738	0.769	0.776	0.774	0.787

* Partial equilibrium exercise. $\theta = 0.2$ but we keep the interest rate constant at the benchmark level $r = 4.63\%$.

Table 10: FINANCIAL ASSETS AND WEALTH COMPOSITION FOR DIFFERENT DOWN PAYMENTS

	Wealth Quintiles				
θ	1st	2nd	3rd	4th	5th
FINANCIAL ASSETS (share %)					
1	0.00	0.01	2.02	4.88	93.09
0.5	-2.42	-1.46	-0.48	1.93	102.43
0.3	-3.42	-2.52	-0.72	2.06	104.60
0.2	-2.87	-2.24	0.09	3.72	101.31
WEALTH COMPOSITION ($D/(D+K)$)					
1	100.00	98.63	83.85	75.71	48.61
0.5	200.00	190.53	109.05	93.14	46.01
0.3	333.33	300.59	113.47	94.02	46.69
0.2	500.00	377.26	92.82	89.47	43.86

Table 11: PRECAUTIONARY SAVINGS FOR DIFFERENT DOWN PAYMENTS

Down Payment, θ	1	0.5	0.3	0.2
Y	1.31	1.29	1.20	1.30
$\Delta(K+D)$	124.50	118.71	115.51	122.67
$\Delta(K+(1-\theta)D)$	320.53	155.38	132.63	134.34

stock). Thus, we infer from this result that the implied volume of precautionary savings is lower when there is durability in consumption.

This result is consistent with the findings of Díaz, Pijoan-Mas, and Ríos-Rull (2000). There, the authors report that precautionary savings are higher when preferences are subject to habit formation, since habits are the opposite to durability (see Ferson and Constantinides 1991). The literature defines habits as a weighted sum of past consumption expenditures and habit formation mean that current utility depends negatively on the volume of past consumption expenditures. In other words, habit formation implies that current consumption expenditures negatively impact future utility. This in turn implies that the loss of utility induced by a bad earnings shock is larger the larger the volume of past consumption expenditures. Thus, individuals fear variations in consumption more than in an economy without habits and consequently, precautionary savings are higher. On the contrary, durability means that current utility depends positively on the volume of past consumption expenditures. In other words, current consumption expenditures positively affect future utility. The loss of utility induced by a bad earnings shock is lower the larger the volume of past consumption expenditures, even if no collateralized borrowing is allowed. Thus, households can bear higher variations in consumption expenditures and do not accumulate as much savings as in the case in which there is no durability.

7 Final comments

In this paper, we have shown that a heterogeneous agents model with uninsurable idiosyncratic risk that explicitly considers the role of durable goods and collateral lending can account for the wealth distribution and wealth composition of US households, as long as the earnings process is fairly persistent.

The two-asset model presented here has one main advantage over the standard one-asset framework previously used to study wealth distribution issues, since it allows to study wealth composition as well. It is important to stress as well that the two-asset model predicts a lower increase in precautionary savings due to uncertainty than the one-asset counterpart.

This paper is one of the few papers that model durables explicitly in a heterogenous agent framework, and to our knowledge, is the only one that incorporates adjustment costs in the durables market.

In this study we have abstracted from some important issues that remain topics for future research. These include incorporating a rental market into the framework to determine if the results presented here in terms of wealth composition are robust to the existence of these markets. Moreover, one could consider the impact of life-cycle effects. It would be also interesting to explore the portfolio composition of households of different ages and analyze whether or not the model can account for life-cycle patterns of wealth holding and wealth composition.

Appendices

A Calibration

In this Appendix we describe in detail the calibration of the share of capital in each model economy.

A.1 National Accounts

In our calibration, we use data from the *National Income and Product Accounts* (NIPA) and *Fixed Reproducible Wealth* published by the Bureau of Economic Analysis for the years 1954-1999. This section discusses the relationship between the one-asset and the two-asset economies and the corresponding NIPA categories.

Denote measured GDP as follows:

$$(c + sh + i_{cd}) + (i_{prk} + i_{pnrk}) + g + nx + \Delta inv = GDP, \tag{9}$$

Expression (9) presents final expenditure components of GDP, where we have grouped categories that belong to the same type in parentheses. $(c + sh + i_{cd})$ are expenditures on private consumption. They are comprised of expenditures on nondurable and services excluding housing (c), housing services (sh), and expenditures on consumer durable goods (i_{cd}). The category $(i_{prk} + i_{pnrk})$ is total private investment, which is the sum of residential and nonresidential private investment. The other categories are public expenditures, g, net exports, nx, and the change in inventories, Δinv.

We can also write output as value added:

$$GDP = we + r_{prk}prk + r_{pnrk}pnrk \tag{10}$$

GDP is equal to the sum of wages plus rents of residential and nonresidential private stocks of capital. Of course, it should be the case that $sh = r_{prk}prk$.

A.1.1 The economy with only one asset

In order to properly calibrate a model of heterogeneous agents with only one asset and without taxes and government expenditure, we have to make some appropriate imputations. First, we should treat expenditures on consumer durables as part of total investment. Second, we must add public expenditure to consumption. Third, we should augment investment with net exports since we are assuming a closed economy. Therefore:

$$(c + sh + g) + (i_{prk} + i_{pnrk} + i_{cd} + nx + \triangle inv) = GDP.$$

Since services of consumer durables, $r_{cd}cd$, do not appear in measured GDP, we impute them to GDP and our measure of output is (from the perspective of expenditure and value added) becomes:

$$(c + sh + g + r_{cd}cd) + (i_{prk} + i_{pnrk} + i_{cd} + nx + \triangle inv) = we + r_{prk}prk + r_{pnrk}pnrk + r_{cd}cd.$$

The share of capital is:

$$\text{share of capital } = \frac{r_{prk}prk + r_{pnrk}pnrk + r_{cd}cd}{we + r_{prk}prk + r_{pnrk}pnrk + r_{cd}cd}.$$

Note also that the implied interest rate is a weighted average of the returns of residential assets, non residential assets and consumer durable goods.

A.1.2 The economy with two assets

Now, we want to explicitly consider the existence of durable goods that comprise consumer durable goods and residential assets. Thus, we define a new category, durables, $d = prk + cd$. Consequently, investment in durables is $i_{pnrk} + i_{cd}$. Rearranging:

$$(c + g) + (sh + r_{cd}cd) + (i_{pnrk} + nx + \triangle inv) + (i_{prk} + i_{cd}) = we + r_{pnrk}pnrk + (r_{prk}prk + r_{cd}cd).$$

Investment in capital is nonresidential investment, and investment in durables is the sum of residential investment and expenditure on consumer durable goods.

Let us now define our measure of output taking into account several considerations. First, in the aggregate, $sh + r_{cd}cd = r_{prk}prk + r_{cd}cd$. Second, services of durable goods are considered final consumption. From this point of view, we should include it as another category in consumption and keep the rental income of durables, $(r_{prk}prk + r_{cd}cd)$, as part of value added. However, since there is not a rental market for durable goods in our model economy, there is not a price for durable services and we can only define its opportunity cost. The absence of this rental market dictates our definition of output in the model:

$$(c + g) + (i_{pnrk} + nx + \triangle inv) + (i_{prk} + i_{cd}) = we + r_{pnrk}pnrk. \tag{11}$$

In order to represent value added as a Cobb-Douglas function of labor and capital (non residential stock of capital), we need to calibrate the share of capital as:

$$\text{share of capital } = \frac{r_{pnrk}pnrk}{we + r_{pnrk}pnrk}.$$

Thus, the definition of output and the share of capital differ depending on which model economy we calibrate. Note that the economy with just one asset could be thought of as an economy with a perfect rental market for durable goods that eliminates the friction of the down payment. In such an economy, the distribution of wealth would be invariant with respect to the composition of the household's portfolio.

A.2 The One-Asset Economy

A question that we have not addressed above is whether we should include government owned capital as part of aggregate capital and, consequently, augment measured GDP with the imputed flow of services of this capital. Since the focus of this paper is on the distribution of privately owned wealth, we exclude government owned capital from our analysis. We construct broad measures of output, Y, investment, I, and aggregate capital, K, according to the organization discipline described before:

$$\begin{aligned} Y &= we + r_{prk} prk + r_{pnrk} pnrk + r_{cd} cd, \\ K &= PRK + PNRK + CD + INV, \\ I &= i_{prk} + i_{pnrk} + i_{cd} + nx + \triangle inv. \end{aligned} \tag{12}$$

To construct an appropriate measure of output, we impute the flow of services of durable goods as in Cooley and Prescott (1995). Income from capital is related to the stock of capital by the following expression:

$$Y_{K_p} = \left(i + \delta_{K_p}\right) K_p, \tag{13}$$

where Y_{K_p} is income of private fixed capital, $r_{prk} prk + r_{pnrk} pnrk$, and δ_{K_p} is the depreciation rate of that capital stock. Given measured values of the capital stock, for the capital income and a measured value for depreciation, we can obtain an estimate of i, the return to capital. For our analysis, measured K_p includes the net stock of private capital (not including the stock of consumer durable goods), and the stock of inventories, $PRK + PNRK + INV$. Both measures are taken from the *Fixed Reproducible Wealth* study.

There are categories of income in NIPA that are unambiguously capital income–rental income, corporate profits and net interest–whereas some other categories, such as proprietors's income cannot clearly be imputed to capital in full. Define *unambiguous capital income* (UCI) and *ambiguous capital income* (ACI) as follows:

$$\begin{aligned} UCI &= \text{Rental Income + Corporate Profits + Net Interest}, \\ ACI &= \text{Proprietors Income + Net National Product - National Income}. \end{aligned}$$

Denote Y_{K_p} as:

$$Y_{K_p} = UCI + \theta ACI + \text{Depreciation} \ = \theta GNP,$$

where Depreciation is consumption of Fixed Capital, $\delta_{K_p} K_p$. This equation can be solved for θ,

$$\theta = \frac{UCI + \text{ Depreciation}}{GNP - ACI}.$$

The value of Y_{K_p} is obtained as θGNP and $i = Y_{K_p} / K_p - \delta_{K_p}$. The average estimated value of the return to capital for the period 1954-99 is 8.42%. We then use this estimated return to calculate services from the stock of consumer durable goods, $r_{cd}cd$, given the measured value of the stock of consumer durables and the measured value of depreciation.

We have defined output as measured GDP plus the estimated value of the flow of services of consumer durable goods. The stock of capital is the sum of private fixed assets, the stock of inventories and the stock of consumer durables. Therefore, the share of capital is then computed as the ratio:

$$\alpha = \frac{Y_{K_p} + r_{cd}cd}{GDP + r_{cd}cd}.$$

The estimated value for the period 1954-1999 is 34.8 percent.

With the definitions shown in (12), we can compute an average capital-output ratio for the period mentioned equal to 2.5061 and an investment-capital ratio equal of 0.0844. Note that with a Cobb-Douglas technology, in a steady state, the interest rate implied by these numbers is

$$r = \alpha \frac{Y}{K} - \frac{I}{K} = 5.24\%.$$

A.3 The Economy with Two Assets

The explicit consideration of durable goods requires to redefine all aggregate variables:

$$\begin{aligned} Y &= we + r_{prk}prk, \\ K &= PNRK + INV, \\ D &= PRK + CD, \\ I_k &= i_{pnrk} + nx + \triangle inv, \\ I_d &= i_{prk} i_{cd}. \end{aligned} \tag{14}$$

As before, we follow Cooley and Prescott (1995), but now we subtract from capital income the item of housing services, sh, and Y_{K_p} is just equal to $r_{prk}prk$. Thus, we have:

$$Y_{K_p} = UCI - sh + \theta ACI + \text{Depreciation } = \theta \left(GNP - sh \right).$$

Proceeding as before:

$$\alpha = \frac{Y_{K_p}}{GDP - sh}.$$

Note now that the share of capital is equal to θ, which depends on the difference between GDP and GNP. The estimated share of capital is 0.18639.

With the definitions shown in (14), we compute an average capital-output ratio $K/Y = 1.4673$, an investment-capital ratio $I_K/K = 0.0814$, a durable stock to output ratio,

$D/Y = 1.4384$, an investment-durable stock ratio, $I_D/D = 0.0913$ and a nondurable to durable investment ratio, $C/I_D = 5.7623$. Note that with a Cobb-Douglas technology, in a steady state, the interest rate implied by these numbers is:

$$r = \alpha \frac{Y}{K} - \frac{I_K}{K} = 4.63\%.$$

B Data on Distribution

We use data from the 1998 *Survey of Consumer Finances*, (hereafter SCF98). Our variable Total Wealth corresponds to what the SCF98 calls net worth. The data on durable goods corresponds to the reported value of three items: value of vehicles, (variable VEHIC), value of residential assets (variable HOUSES) and other residential assets, (ORESE). Thus, consistent with our definition of durables in our model economy, the value of durables is always non-negative.[10]

C Computational Procedures

C.1 Reformulation of the model

In order to compute the equilibrium of the model, it is convenient to reformulate the household problem. Define voluntary equity, $q = k + (1-\theta)d$, as the wealth held in excess of the required down payment. A household's state variables are its earnings shock, its holdings of voluntary equity, and its holdings of durable good, $\{e, q, d\}$. In this way, we have two assets whose values are restricted to be non-negative. This greatly simplifies the problem imposed by the endogenous liquidity constraint in the solution of the household problem. Define Q as aggregate voluntary equity. We write feasibility as a function of this new variable,

$$C + Q' - (1-\delta_k)Q + \theta D' + \left[(1-\delta_k)(1-\theta) - (1-\delta_d)(1-I\rho)\right] D = F\left(Q - (1-\theta)D, L\right).$$

Factor prices can also be written as functions of Q and D.

Note that our individual state space contains two endogenous individual variables (voluntary equity and durable assets) as well as the exogenous idiosyncratic shock. This implies the need to create a two dimensional grid for the two endogenous states. We use convex grids and do not interpolate between grid points.

[10] There is, however, an inconsistency. In the aggregate data we have information on the total stock of durables (housing, vehicles and other durable goods), whereas at the disaggregate level we only have information on real estate and vehicles. Nevertheless, we believe that the discrepancy is not important in quantitative terms.

C.2 Solving the household problem

The household problem can be rewritten as follows:

$$v(e,q,d) = \max_{q' \geq 0, d'} \left\{ u(c,d) \; + \; \beta \sum_{e'} \pi_{e,e'} \; v\left(e',q',d'\right) \right\}, \tag{15}$$

where

$$c = we + (1+r)\; q + \left[\left(1-\delta^d\right)(1-I\rho) - (1-\theta)(1+r)\right] \; d - q' - \theta d' \tag{16}$$

Note that by reformulating the model this way, we deal with an exogenous borrowing limit, zero, in voluntary equity, q, instead of the endogenous liquidity constraint in capital, k, shown in (2). This transformation allows us to work with a grid in q that starts at zero, instead of starting at some (endogenous) negative number, which would be required if we worked with the original state variable k. The grid for the durable good also starts at zero since we have assumed that marginal utility of durable goods at zero was positive. The upper values for the grids must be chosen with care.

We solve this problem by value function iteration. We guess an initial value function v^0, and make agents choose next period's values of q' and d' in the grid. In principle, choosing values of the policy function in the grid may be very costly computationally in a heterogeneous agents model. We avoid this problem by using a policy function accelerator described in Judd (1998):

1. Given the initial guess of the value function v^0 obtain the associated policy functions choosing their values within the grid, $g_0^q(e_i, q_j, d_l)$ and $g_0^d(e_i, q_j, d_l)$.

2. Iterate a sufficiently large number of times on the value function using g_0^q and g_0^d. This is the accelerator. Call the resulting function v^1. If v^1 and v^0 are close enough we reached the fixed point. The associated policy functions are those that solve the household's problem. If they are not, rename $v^0 = v^1$ and go back to step 1.

By the N-Stage Contraction Theorem, there is a unique fixed point v that solves the household problem. Moreover, the speed of convergence is much greater than in the regular case, which allows us to use a grid for both assets of 80×80 points. This implies solving the household's problem for 19,200 points at each iteration

C.3 Solving for the Steady State

We outline the algorithm below.

Step 1. For an initial guess of the aggregate stock of durables, D_0, we choose an initial guess Q_0 (so that the interest rate is less than the discount factor and durables depreciation factor) and compute the implied factor prices $\{r_0, w_0\}$.

Step 2. Given those factor prices, we solve the household problem to get the associated policy functions g_0^q and g_0^d.

Step 3. We find the aggregate voluntary equity, Q_1, and the aggregate stock of durables D_1 implied by individual optimal saving behavior at the given factor prices $\{r_0, w_0\}$. To do this, we guess an initial sample of individuals of size 9,000 and apply to them g_0^q, and g_0^d and the earnings Markov process 3,000 times, which ensures in all experiments that the main statistics of the sample are almost constant. To ensure no sampling error is spoiling the convergence in Q, we use the same seed to initialize the random number generator in each iteration. If Q_0 and Q_1 are sufficiently close, stop. Otherwise, update $Q_0 = (1-\varepsilon)Q_0 + \varepsilon Q_1$ and go back to step 1 *without changing the initial guess* D_0. Once we have found a fixed point for Q, we have an aggregate stock of durables $D_1(D_0)$.

Reformulating the model this way is especially advantageous at this step. The reason is the following: Aggregate voluntary equity is a non-negative function of the interest rate, whereas aggregate capital, K, is not. Aggregate capital becomes negative for sufficiently small levels of the interest rate and is very elastic. Hence, convergence in K may be slow since aggregate capital may overshoot with small changes in interest rate. This is not the case for aggregate voluntary equity.

Step 4. Once Q_1 has converged, update $D_0 = D_1(D_0)$ and repeat step 3 until a fixed point for D is found.[11]

[11] In practice, three iterations for step 4 were sufficient to find convergence of D.

References

ABOWD, J. M., AND D. CARD (1987): "Intertemporal Labor Supply and Long Term Employment Contracts," *American Economic Review*, 77, 50–68.

——— (1989): "On the Covariance Structure of Earnings and Hours Changes," *Econometrica*, 57(2), 411–45.

AIYAGARI, S. R. (1994): "Uninsured Idiosyncratic Risk, and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

ALESSIE, R., M. P. DEVEREUX, AND G. WEBER (1997): "Intertemporal Consumption, Durables and Liquidity Constraints: A Cohort Analysis," *European Economic Review*, 41, 37–59.

BRUGIAVINI, A., AND G. WEBER (1994): *Saving and the Accumulation of Wealth: Essays on Italian Household and Government Saving Behavior*chap. Durables and Non-Durables Consumption: Evidence from Italian Household Data. Cambridge University Press, Cambridge.

CAMPBELL, J. Y., AND A. DEATON (1989): "Why is consumption so smooth," *Review of Economic Studies*, 56, 357–374.

CARROLL, C. D. (2000): "Why Do the Rich Save So Much?," in *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, ed. by J. B. Slemrod. Harvard University Press.

CASTAÑEDA, A., J. DÍAZ-GIMÉNEZ, AND J. V. RÍOS-RULL (2001): "Accounting for earnings and wealth inequality," Mimeo, University of Pennsylvania.

CHAH, E. Y., V. A. RAMEY, AND R. M. STARR (1995): "Liquidity Constraints and Intertemporal Consumer Optimization: Theory and Evidence from Durable Goods," *Journal of Money, Credit, and Banking*, 27(1), 272–287.

COOLEY, T. F., AND E. C. PRESCOTT (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, chap. 1. Princeton University Press, Princeton.

DÍAZ, A., J. PIJOAN-MAS, AND J.-V. RÍOS-RULL (2000): "Habit Formation: Implications for the Wealth Distribution," Mimeo University of Pennsylvania.

DÍAZ-GIMÉNEZ, J., AND L. A. PUCH (1998): "Borrowing Constraints in Economies with Indivisible Household Capital and Banking: an Application to the Spanish Housing Market (1982-89)," *Investigaciones Económicas*, 22(3), 469–499.

FARR, C. C., AND M. J. LUENGO-PRADO (2001): "The Implications of Lower Down Payments on COnsumption Volatility," Manuscript, Universidad Carlos III de Madrid.

Fernández-Villaverde, J., and D. Krueger (2001): "Consumption and Saving Over the Life Cycle: How Important are Consumer Durables?," Manuscript, Stanford University.

Ferson, W. E., and G. Constantinides (1991): "Habit Persistence and Durability in Aggregate Consumption," *Journal of Financial Economics*, 29(2), 199–240.

Flavin, M. (1989): "The adjustment of consumption to changing expectations about future income," *Journal of Political Economy*, 89, 974–1009.

Grossman, S. J., and G. Laroque (1990): "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Conusmption Goods," *Econometrica*, 58(1), 25–51.

Hopenhayn, H., and E. C. Prescott (1992): "Stochastic Monotonicity and Stationary Distributions for Dynamic Economies," *Econometrica*, 60, 1387–1406.

Huggett, M. (1993): "The Risk Free Rate in Heterogeneous-Agents, Incomplete Insurance Economies," *Journal of Economic Dynamics and Control*, 17(5/6), 953–970.

——— (1996): "Wealth Distribution in Life-Cycle Economies," *Journal of Monetary Economics*, 38(3), 469–494.

J. Díaz-Giménez and E. C. Prescott and T. Fitzgerald and F. Álvarez (1992): "Banking in Computable General Equilibrium Economies," *Journal of Economic Dynamics and Control*, 16, 533–559.

Judd, K. L. (1998): *Numerical Methods in Economics.* MIT Press.

Krusell, P., and A. Smith (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106, 867–896.

Kydland, F. E. (1984): "Labor-Force Heterogeneity and the Business Cycle," *Carnegie-Rochester Conference Series on Public Policy*, 21, 173–209.

Quadrini, V., and J.-V. Ríos-Rull (1997): "Understanding the U.S. Distribution of Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 21, 22–36.

Ríos-Rull, J.-V. (1995): "Models with Heterogenous Agents," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, chap. 4. Princeton University Press, Princeton.

Tauchen, G. (1986): "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20, 177–181.