# Durables, Nondurables, Down Payments and Consumption Excesses*

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## Abstract

We examine a model that generalizes the standard buffer-stock model of savings to accommodate durables, nondurables and a collateralized liquidity constraint, with and without adjustment costs in the durables market. Since there is no known analytical solution to the model, we solve it numerically. We find that nondurable consumption becomes more volatile as down payment requirements decrease at the individual and at the aggregate level. Moreover, for plausible parameter values the model can explain the excess smoothness and excess sensitivity observed in the U.S. economy. The results follow from a gradual adjustment of consumption to permanent income shocks in an attempt by agents to spread out the burden of down payments over time.

KEYWORDS: Buffer stock, Consumption, Durable Goods, Incomplete Markets, Computational Economics.
JEL CLASSIFICATIONS: E21 - Consumption; Saving, C36 - Computational Techniques, C61 - Optimization Techniques; Programming Models; Dynamic Analysis.

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# 1 Introduction

Private consumption is the most important component of aggregate demand. Nevertheless, the modelling of consumption behavior is still a challenge to the economics profession. Indeed, Alan Greenspan recently acknowledged that the Federal Reserve does not have as clear an understanding of consumer spending as they would like.[1] Understanding consumption is essential for the construction of meaningful macroeconometric forecasting models, as well as for evaluating the effects of key fiscal and monetary policies.

One of the lasting contributions to modern day consumer theory is the idea that consumption is determined by the expected value of lifetime resources (the "lifecyle" theory, LC) or permanent income (the PIH), pioneered by Modigliani & Brumberg (1954) and Friedman (1957). The modern-day specification of this fundamental concept is typically thought of as an intertemporal choice model with quadratic preferences, stochastic labor income, no borrowing restrictions and perfect foresight (Hall (1978), Flavin (1981)).

In spite of its intuitive appeal, several influential papers have revealed discrepancies between this model's predictions and the aggregate data. On one hand, the LC-PIH predicts that consumption should be a martingale, in particular, current innovations to consumption should be independent of past innovations to income (Hall (1978)). However, the aggregate evidence overwhelmingly rejects this proposition as the correlation between consumption growth and lagged income growth is a very robust feature of the aggregate data (e.g. Flavin (1981), Blinder & Deaton (1985), Campbell & Deaton (1989), Attanasio & Weber (1993)). This empirical regularity is known in the literature as the *excess sensitivity* of consumption. On the other hand, the LC-PIH also predicts that if the income process exhibits high persistence, current consumption should respond strongly to unanticipated innovations in disposable income, since an innovation to current income results in a large innovation to discounted expected future income as well. However, aggregate consumption is in fact much smoother than aggregate income (e.g. Deaton (1987), Campbell & Deaton (1989), a fact referred to in the literature as the *excess smoothness* of consumption.

Researchers have worked on modifications of the LC-PIH framework in an attempt to reconcile theory and empirical evidence. The buffer stock model of savings, pioneered by Deaton (1991) and Carroll (1997), has become the standard model for thinking about the way a median consumer behaves. This model introduces precautionary motives for saving, impatience and borrowing restrictions into the standard LC-PIH framework. Over the last

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[1] "Consumer Spending Puzzles the Fed: Old Formulas Based on Income are Inadequate at Predicting Behavior", *International Herald Tribune*, September 3, 2001.

decade a large body of literature has shown that this model can explain well several aspects of household spending decisions. However, at the aggregate level, the implications of the buffer stock model are not fully satisfactory. Ludvigson & Michaelides (2001) show – in a careful and explicit aggregation of the buffer stock model of savings – that this model cannot generate robust "excesses". Instead, they must rely on incomplete information à la Pishcke (1995) to generate some excesses. Similarly, Michaelides (2001) needs to add habit formation to the basic framework to replicate the excesses observed in the aggregate data.

We believe that one main shortcoming of most consumption models, including the buffer stock model, is that they have traditionally focused on the study of nondurable consumption alone.[2] In this paper, we present a generalized buffer stock model of savings with durables and nondurables together to show that such a model could explain the excess sensitivity and excess smoothness of nondurable consumption observed in the aggregate data. Introducing durable goods into this framework is not straightforward and it can be done in several different ways. We propose the following approach. We begin with the classic buffer stock framework. However, we assume that agents derive utility from consumption of a nondurable good *and* the services obtained from holding a durable good. Moreover, the durable stock can act as collateral for credit purchases. This means that after satisfying a down payment requirement, the remaining fraction of the durable value can be financed. This kind of constraint imposes distortions on the allocation of consumption across time and across goods, even with a utility function separable in both goods, which we assume.[3] Finally, we take into account the fact that the market for durables may be characterized by important transaction costs. We consider non-convex costs of adjustment as in Grossman & Laroque (1990), which generate large and infrequent adjustments, in a $(S, s)$ rule fashion.[4] In all other respects, our model is identical to the classic buffer stock framework.

Considering a collateralized liquidity constraint should not be controversial. According to the Federal Reserve Board's 1995 *Survey of Consumer Finances* (SCF), collateral borrowing, mainly obtained to purchase housing and automobiles, is the principal type of borrowing undertaken by households.[5] In many cases, consumers must meet a certain

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[2] There are of course exceptions. For example Caballero (1993), Eberly (1994), Chah, Ramey & Starr (1995), Carroll & Dunn (1997), Dunn (1998).

[3] Even without collateralized liquidity constraints, the interactions between durable and nondurable goods are interesting. Browning & Crossley (1997) find evidence of individuals who face limited borrowing alternatives smooth out fluctuations in income by postponing replacement of small durables.

[4] See Attanasio (1998) for more references and an insightful discussion on models with lumpy adjustment.

[5] In the survey, 65.2% of total borrowing was attributed to the purchase of homes, and 7.1% of the total

down payment requirement before qualifying for collateral lending.[6] Moreover, required down payments represent a large financial burden for most households. For example, in the housing market, according to "Who's Buying Homes in America" (an annual survey of home buyers), households must save, on average, for two and half years to buy their first home.[7] For first-time buyers, the majority of the required down payment comes from savings (74.8 percent). For repeat buyers, savings are supplemented by the proceeds from the existing house sale (52.2 percent from savings and 31.7 percent from the sale of the previous home). Note that considering this constraint allows us to consider an extra motive for savings: saving for down payments. However, it does not imply a fixed borrowing limit, as in Ludvigson & Michaelides (2001) and Carroll (1997), but a limit that varies with the durable stock. Ludvigson (1999) studies a buffer stock model of savings with time varying liquidity-constraints, where credit varies stochastically with income. In the framework we propose, credit availability could change over time through changes in down payment requirements.

It is important to acknowledge that a very similar formulation of the problem *without* adjustment costs has been explored by Chah et al. (1995), Alessie, Devereux & Weber (1997) and Brugiavini & Weber (1994). However, the focus of these papers is empirical. To our knowledge, the literature has been unable to solve this model due to the *curse of dimensionality* and the complications that the illiquidity of the durable poses. We show that with a combination of the right tricks and numerical dynamic programming techniques, the dimensionality of the problem can be overcome and reasonable parameterizations of the model can be solved accurately. In particular, we use Euler equation iteration for a version of the model with no adjustment costs in the durable market and a finite state approximation method for the version with adjustment costs. Both techniques are discussed and compared briefly.[8]

In this paper, we characterize the optimal consumption rules for an individual consumer under different down payment regimes. We then simulate individual and aggregate consumption series – calculated through explicit aggregation – and study the implications

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debt was used to buy vehicles. Total borrowing is measured as the dollar amount of all debt reported by respondents. See Kennickell, Starr-McCluer & Sunden (1997).

[6] In addition to the collateral requirement, lenders impose several additional criteria to reduce the likelihood of default. For housing, some lenders require that the mortgage payment does not exceed some percentage of current income. Another standard condition requires the loan-to-value ratio (LTV) to be below a certain threshold. Otherwise, the borrower faces higher marginal borrowing costs, including a higher interest rate and the purchase of mortgage insurance.

[7] See Chicago Title and Trust (1995).

[8] For a more detailed and formal discussion see Farr & Luengo-Prado (1999).

of different down payments for consumption patterns. Finally, we explore if a plausible parametrization of the model can account for the excess sensitivity and excess smoothness observed in the aggregate data for nondurable consumption. This is done first for the model without adjustment costs of changing the durable stock. The robustness of the results is verified by introducing adjustment costs.

At the individual level, we show that it is very important to distinguish between transitory and permanent shocks. We find that in the case with no-adjustment costs, when income is transitorily low, a buffer stock agent on occasion liquidates the equity accumulated in the durable implied by the down payment requirement to prop up his nondurable consumption. This effect implies that higher required down payment regimes – which translate into higher levels of equity – are associated with smoother individual nondurable consumption patterns, a result consistent with Carroll & Dunn (1997). On the other hand, we also find that when an agent experiences a (positive) permanent income shock, he chooses not to fully adjust his consumption (durable and nondurable) due to the his desire to spread out the cost of accumulating a down payment. This also results in smoother consumption under higher down payment regimes.

At the aggregate level, we also show that nondurable consumption becomes smoother as down payment requirements increase. This results from the gradualism of consumption induced by the desire of agents to spread out the cost of the down payment, not the higher equity levels associated with higher down payments. This is due to the fact that at the aggregate level, all idiosyncratic shocks cancel out and all that remains is the effect of aggregate shocks which are modelled as permanent. This is also consistent with the empirical evidence. Given the sluggish response of consumption to changes in income, the model can generate robust excesses for reasonable parameter values.

The implications for the durable are slightly more complex and are thoroughly discussed throughout the paper. In general, all results are robust to the inclusion of adjustment costs. Additionally, our results are consistent with the international empirical evidence in Japelli & Pagano (1989), who found that "the excess sensitivity of consumption to current income fluctuations is higher in countries where consumers borrow less", in particular down payment requirements for home mortgages are highly correlated with excess sensitivity.

The remainder of the paper is organized as follows. Sections 2 and 3 introduce the agent's problem when there are no costs of adjustment, discuss the solution method, and describe the optimal policy functions for nondurable and durable consumption. In section 4, we present our baseline calibration and discuss the simulation implications of the model for consumption patters both for an individual household and for the aggregate. Section

5 presents the model with adjustment costs. Section 6 provides concluding remarks.

## 2 The Basic Model

The consumer's problem is to maximize the present discounted value of expected utility from consumption of a nondurable good, $C_t$, and a durable good, $K_t$, where $t$ denotes time.[9] We will assume that time is discrete and agents face an infinite horizon. $\beta < 1$ is their discount rate.

$$\max_{\{C_t,K_t\}} V = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \, U(C_t, K_t) \right\} . \tag{1}$$

The instantaneous utility function is assumed to be separable in both goods and is of the CRRA type:

$$U(C_t, K_t) = \frac{C_t^{1-\rho}}{1-\rho} + \varphi \frac{K_t^{1-\rho}}{1-\rho} , \tag{2}$$

where $\varphi$ is a preference parameter.[10] Note that $\rho > 0$ implies that the agent is risk-averse and has a precautionary motive for saving.

This maximization is subject to both a budget constraint and a liquidity constraint. Assume there is one riskless financial asset, $A_t$. $R$ is the interest factor paid on it. In period $t$, an agent holds past financial assets gross of interest, $RA_{t-1}$. He also receives $Y_t$ units of income. In the same period, the agent chooses nondurable consumption, $C_t$, and net investment on the durable, $(K_t - \psi K_{t-1})$, where $\psi$ is the depreciation factor of the durable good. Therefore, the budget constraint between two successive periods is given by

$$A_t = RA_{t-1} + Y_t - (K_t - \psi K_{t-1}) - C_t \; . \tag{3}$$

Labor income is assumed to be exogenous to the agent and stochastic, and it is the only source of uncertainty in the model. We will assume, as in Ludvigson & Michaelides (2001) and similar to Carroll (1997), that:

$$Y_t = P_t V_t,$$

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[9] We make the simplifying assumption that the agent's service flows from the durable are proportional to the durable stock held and set the constant of proportionality equal to one. This simplification has been used by many others. See for example Mankiw (1982) or Chah et al. (1995). A slightly more realistic setup would express utility as a function of the service flows derived from the durable stock. These services could be affected by, among other things, frequency of use.

[10] We follow Bernanke (1984) who studied the joint behavior of the consumption of durable and nondurable goods and found that separability was a good approximation. With regards to prices, we assume that $P_t^C / P_t^K = 1$, $\forall t$.

$$P_t = G_t P_{t-1} N_t.$$

Labor income $Y_t$ is the product of permanent income, $P_t$, and an idiosyncratic transitory shock, $V_t$. $G_t$ can be thought of as the growth in permanent income attributable to aggregate productivity growth in the economy which is common to all agents. $N_t$ is a permanent idiosyncratic shock. We assume that $\ln V_t$ and $\ln N_t$ are independent and identically distributed with mean zero and variances $\sigma_V^2$ and $\sigma_N^2$, respectively. $\ln G_t$ has mean $\mu_G$ and variance $\sigma_G^2$. This specification implies that the growth rate of individual labor income follows an MA(1) process,

$$\Delta \ln Y_t = \ln G_t + \ln N_t + \ln V_t - \ln V_{t-1},$$

which is consistent with the microeconomic evidence.[11]

The aggregate implications of the process are also consistent with aggregate labor income for an appropriate choice of parameters. It is not difficult to show that if individual income follows the process above, aggregate income, $\bar{Y}_t$, follows the process,

$$\Delta \ln \bar{Y}_t = \ln G_t + 0.5\sigma_N^2,$$

where $\bar{Y}_t = 1/n \sum_{i=1}^{n} Y_{it}$, $i = 1, \ldots, n$.

A very important aspect of the model is the collateralized liquidity constraint imposed on the agent:

$$A_t + (1-\theta)K_t \geq 0 \ , \tag{4}$$

with $\theta \in [0,1]$. An individual's borrowing limit is a function the durable stock, since this can act as collateral for credit purchases. Alternatively, the constraint can be written as $A_t + K_t \geq \theta K_t$, which states that there is a lower bound to net worth (financial assets plus physical assets) given by a proportion $\theta$ of the durable stock. $\theta$ can be interpreted as a down payment parameter. Plugging (3) into the liquidity constraint reveals how the two goods are treated differently:

$$C_t + \theta K_t \leq RA_{t-1} + \psi K_{t-1} + Y_t \ . \tag{5}$$

Only a down payment of the durable, given by $\theta K_t$, is relevant in the liquidity constraint.

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[11] See the discussion in Ludvigson & Michaelides (2001) on how empirical studies such as MaCurdy (1982), Abowd & Card (1989), and Pishcke (1995), find that an MA(1) in the growth rate of income is a good approximation to models estimated on microeconomic data.

At the extremes, $\theta = 0$ would imply that durable purchases do not affect current liquidity. When $\theta = 1$, the liquidity constraint imposed upon the agent treats expenditure on both goods equally.

For a preliminary look on how changes in the possibilities of financing the durable good affect nondurable consumption, we rewrite equation (3) to get an expression for net worth,

$$A_t + K_t = RA_{t-1} + \psi K_{t-1} + Y_t - C_t \ .$$

Differencing the equation above and rearranging yields:

$$C_t = (R-1)A_{t-1} + (\psi - 1)K_{t-1} + Y_t - \Delta(A_t + K_t) \ .$$

Define $Q_t \equiv A_t + (1-\theta)K_t$ to be *voluntary equity*, the wealth held in excess of the required down payment, $\theta K_t$.[12] The change in net worth is given by:

$$\Delta(A_t + K_t) = \Delta(A_t + (1-\theta)K_t + \theta K_t) = \Delta Q_t + \theta \Delta K_t \ .$$

The equation above states that we can decompose the change in net worth into two parts: The change in voluntary equity, $Q_t$, and the change in required equity, $\theta K_t$. Changing required equity implies changing the durable stock, while in order to change voluntary equity an agent can simply increase debt relative to his durable stock. Finally, nondurable consumption can be rewritten as:

$$C_t = (R-1)A_{t-1} + (\psi - 1)K_{t-1} + Y_t - \Delta Q_t - \theta \Delta K_t \ . \tag{6}$$

An agent can increase nondurable consumption by decreasing either voluntary or required equity. Different financing regimes for the durable good, which in this model means different $\theta$s, will affect the availability of this last channel.

## 2.1 A Convenient Reformulation of the Model

A closed-form solution of the model developed in last section does not exist and therefore we must rely on computational methods to solve the model. *Euler Equation Iteration* has been the traditional approach for solving microeconomic dynamic stochastic optimization problems with nondurable consumption only. We generalize the algorithm in Carroll

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[12] Note that the liquidity constraint can also be expressed as $Q_t \geq 0$.

(1997) and Deaton (1991) to accommodate multiple goods and the collateralized liquidity constraint considered here. We present a reformulation of the model that facilitates the implementation of this numerical technique as well as the first order conditions of the problem. Appendix A.1 covers further details about the solution technique.

Define cash-on-hand, $X_t$, as $X_t \equiv RA_{t-1} + \psi K_{t-1} + Y_t$. Note that durable wealth can be lumped together with financial resources and labor income because we assume for now that there are no costs of adjusting the durable stock. The budget constraint becomes $A_t = X_t - C_t - K_t$ and the liquidity constraint $C_t + \theta K_t \leq X_t$. Combining the definition of cash-on-hand and the budget constraint we can write an expression for the evolution of cash-on-hand: $X_{t+1} = R(X_t - C_t) + (\psi - R)K_t + Y_{t+1}$. We can then set up the Bellman equation of the problem,

$$\begin{aligned} V_t(X_t) &= \max_{C_t, K_t} U(C_t, K_t) + \beta \mathrm{E}_t \left\{ V_{t+1}[R(X_t - C_t) + (\psi - R)K_t + Y_{t+1}] \right\} \\ &\qquad + \lambda_t \left( X_t - C_t - \theta K_t \right) . \end{aligned}$$

The first-order conditions and the envelope condition yield the following system of equations:

$$U_C^t = \beta R \, \mathrm{E}_t[U_C^{t+1}] + \lambda_t \ , \tag{7}$$

$$\varphi \, U_K^t = \beta (R - \psi) \, \mathrm{E}_t[U_C^{t+1}] + \theta \lambda_t \ , \tag{8}$$

$$\lambda_t (X_t - C_t - \theta K_t) = 0 \ . \tag{9}$$

Equation (7) states that the marginal utility of nondurable consumption in period $t$ must equal the discounted expected marginal utility of nondurable consumption in period $t+1$, plus the shadow price of the liquidity constraint. Analogously, equation (8) states that the marginal utility of durable consumption in period $t$ must be equal to the expected marginal utility of nondurable consumption in period $t+1$ discounted by $\beta(R - \psi)$, plus $\theta$ times the shadow price of the liquidity constraint. Note the difference in the discount factor from the above equation ($\beta R$ versus $\beta(R - \psi)$). $\psi$ in the discount term reflects the durability of $K$. Also, $\lambda_t$ is multiplied by $\theta$ to reflect the fact that only a down payment is required as payment for the durable in period $t$.

Conditions (7) and (8) are intertemporal conditions. Solving for $\beta E_t[U_C^{t+1}]$ in equation (7) and plugging it into equation (8), we can obtain an equation for the intratemporal relationship between $C_t$ and $K_t$:

$$\varphi\, U_K^t = \frac{R-\psi}{R}\, U_C^t + \left(\theta - \frac{R-\psi}{R}\right)\, \lambda_t. \tag{10}$$

When the liquidity constraint is not binding, $\lambda_t = 0$, and for our particular utility function:

$$\frac{C_t}{K_t} = \varphi^{-\frac{1}{\rho}} \left(\frac{R-\psi}{R}\right)^{\frac{1}{\rho}} \equiv \Omega. \tag{11}$$

This is the optimal relationship between $C_t$ and $K_t$ that accounts for durability.[13] $(R-\psi)/R$ is known in the literature as the user cost of the durable. This cost represents the single-period cost, or rental equivalent cost of the agent to purchase a unit of the durable. It is affected by the depreciation factor and the interest rate.[14] When the agent is not liquidity constrained, the trade-off between $C_t$ and $K_t$ is fully captured by the user cost and the preference parameters. For liquidity constrained agents, other factors influence their decisions. If $K_t$ is poor collateral ($\theta$ is higher than the user cost), constrained agents let durable consumption fall temporarily and *vice versa*. Note that when $\theta = (R-\psi)/R$, the trade-off between $C_t$ and $K_t$ is determined only by the user cost, regardless of the constraint binding or not. This is will be a particularly useful benchmark case.[15]

In order to deal with the nonstationarity of income, we normalize all variables by permanent income, $P_t$. Lower-case variables will denote upper case counterparts divided by permanent income. Then, the Euler-Lagrange equations can be rewritten as follows. When the agent is not constrained:

$$\begin{aligned}\beta R\, \mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left( c_{t+1} \left[ (G_{t+1}N_{t+1})^{-1} \left( R(x_t - c_t) + \left(\frac{\psi - R}{\Omega}\right) c_t \right) + V_{t+1} \right] \right)^{-\rho} \right\} \\ -c_t^{-\rho} = 0, \end{aligned} \tag{12}$$

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[13] If $\varphi = 1$ and $\psi$ is 0 (i.e. the durable depreciates completely after one period), $U_C^t = U_K^t$. That is, the agent would choose to consume the same amounts of both goods ($C_t = K_t$). As long as $\psi > 0$, $C_t < K_t$.

[14] Depreciation erodes the agent's investment in the durable and effectively increases its cost. The interest rate also increases the user cost as it reflects the opportunity cost of investing in the durable, a dollar invested in the durable could have gone to the other asset in the model which would have returned $R - 1$ dollars. We ignore other factors, such as capital gains and losses on the durable. A more general specification for the user cost would be $(P_t^K\, R - P_{t+1}^K\, \psi)/(P_t^K R)$.

[15] Note also that equations (7) and (10) imply that

$$R\beta \mathrm{E}_t[U_C^{t+1}] - U_C^t = \left(\theta - \frac{R-\psi}{R}\right)^{-1} \left(U_K^t - \frac{R-\psi}{R} U_C^t\right) .$$

The change in marginal utility between two successive periods is not white noise. This interesting implication of the model was used by Chah et al. (1995) to empirically study the excess sensitivity of consumption.

When the agent is constrained:

$$\beta[\psi - R(1-\theta)]\ \mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left( c_{t+1} \left[ (G_{t+1}N_{t+1})^{-1}[\psi - R(1-\theta)] \frac{x_t - c_t}{\theta} + V_{t+1} \right] \right)^{-\rho} \right\}$$
$$-c_t^{-\rho} + \varphi \left( \frac{x_t - c_t}{\theta} \right)^{-\rho} = 0. \tag{13}$$

The two functional equations above can be used to solve for the policy function of (normalized) nondurable consumption as a function of the only state variable, (normalized) cash-on-hand, $c(x)$. Once we have found the policy function for nondurable consumption, the policy function for durable consumption, $k(x)$, can be backed out by using the intratemporal relationship between the two goods. Details on the derivation of the former equations and the numerical procedure are given in Appendix A.1.

Two sufficient conditions for the individual Euler equations (7) and (8) to define a contraction mapping for $\{c(x), k(x)\}$ are the conditions in Theorem 1 of Deaton & Laroque (1992). In our case:

$$\beta R E_t[(G_{t+1}N_{t+1})^{-\rho}] < 1 \tag{14}$$

$$\beta(R-\psi) E_{t+1}[(G_{t+1}N_{t+1})^{-\rho}] < 1 \tag{15}$$

Using the fact that $G_{t+1}$ and $N_{t+1}$ are independent and taking logs in the above equations we obtain:

$$\frac{1}{\rho}[\ln(\beta) + \ln(R)] + \frac{\rho}{2}(\sigma_G^2 + \sigma_N^2) < \mu_G \tag{16}$$

$$\frac{1}{\rho}[\ln(\beta) + \ln(R-\psi)] + \frac{\rho}{2}(\sigma_G^2 + \sigma_N^2) < \mu_G \tag{17}$$

Equation (16) is the "impatience" condition derived by Deaton (1991) with $\mu_N = 0$. This condition ensures that borrowing is part of the unconstrained plan. For equation (17) to be satisfied, $R > \psi$. Moreover, as long as $0 < R - \psi < 1$, condition (16) is stricter than condition (17). Briefly, for $0 < R - \psi < 1$, the standard impatience condition common to buffer stock models guarantees convergence. For $R - \psi > 1$, convergence is guaranteed by condition (17). For $R < \psi$, convergence is not guaranteed.

## 3 The Policy Functions

We now describe the shape of the optimal rules for normalized durable and nondurable consumption, $k(x)$ and $c(x)$. We compare these rules to the rule for nondurable consumption from the classical buffer stock literature. These functions will help us build some intuition to explain the relationship between down payment requirements and consumption patterns.

In order to compare our findings with the previous literature, we start by adding up $c_t$ and $\theta k_t$. The policy function for this variable for a given $\theta$ is depicted in Figure 1, panel A.

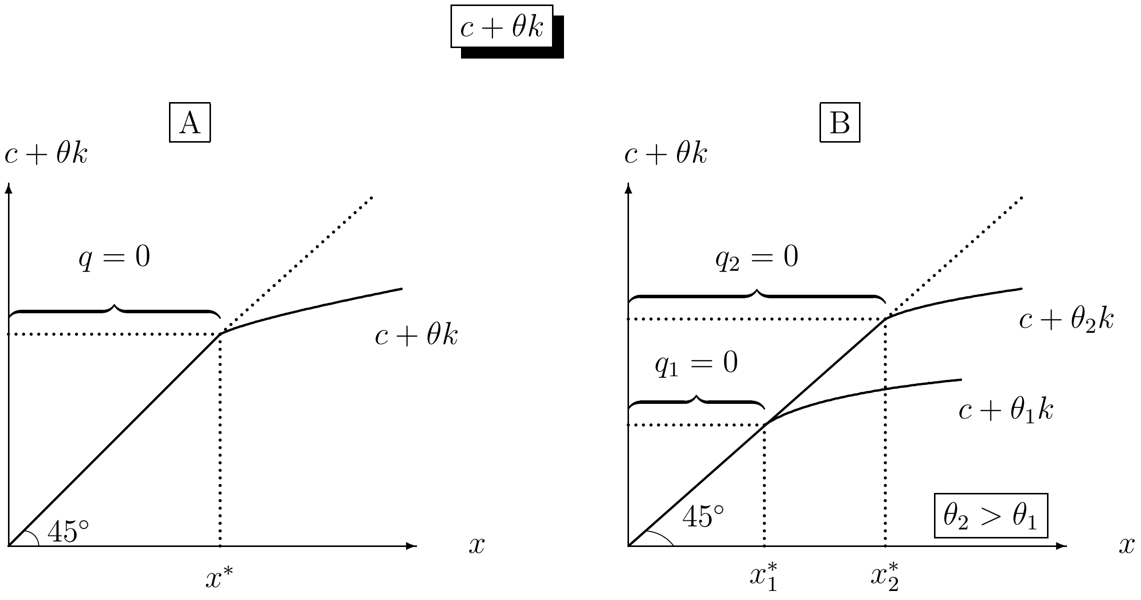

Figure 1: Policy Function. Nondurable plus Down Payment as a Function of Cash-on-Hand.

Similar to Deaton (1991), there is a unique $x^*$ such that:

$$c + \theta k = \begin{cases} x, & x \leq x^*, \\ \\ < x, & x > x^*. \end{cases}$$

If (normalized) cash-on-hand this period, $x_t$, is below a threshold level $x^*$, the agent is liquidity constrained and all resources go to pay for the down payment requirement and nondurable consumption. No (normalized) voluntary equity, $q_t \equiv a_t + (1 - \theta) k_t$, is carried over to the next period. If cash-on-hand is higher than $x^*$, some voluntary equity will be added to current resources the next period. Note that the higher is $\theta$, the higher the required level of equity an agent must keep. We may expect then, that the threshold $x^*$ be an increasing function of $\theta$. This is indeed what we find (see Figure 1, panel B).

We now turn to discuss how resources are allocated between the two goods, since in the interactions of the two goods is where the novelty of the model appears. The key results are summarized in Propositions 1 and 2. Proofs are presented in Appendix B.

**Proposition 1** *When the agent is not liquidity constrained* ($x \geq x^*$),

$$c(x) = \frac{\Omega}{\Omega + \theta}\,(x - q),$$

*and*

$$k(x) = \frac{1}{\Omega + \theta}\,(x - q),$$

*regardless of the value of* $\theta$.

Proposition 1 states that when the agent is not liquidity constrained and once the decision regarding how much voluntary equity to bring to the next period has been taken, the agent spends fixed proportions of the remaining cash-on-hand between the two goods.

**Proposition 2** *When the agent is liquidity constrained* ($x \leq x^*$),

*(A) If no down payment is required (* $\theta = 0$*),*

$$c(x) = x \quad \textit{and} \quad k(x) = \frac{1}{\Omega}\,x^*.$$

*(B) If the down payment parameter is lower than the user cost,* $\theta < \frac{R-\psi}{R}$*,* $c(x)$ *is a convex function of* $x$ *and* $k(x)$ *is a concave function of* $x$*.*

*(C) If the down payment parameter is equal to the user cost,* $\theta = \frac{R-\psi}{R}$*,* $c(x)$ *and* $k(x)$ *are linear functions of* $x$*. In particular:*

$$c(x) = \frac{\Omega}{\Omega + \theta}\,x \quad \textit{and} \quad k(x) = \frac{1}{\Omega + \theta}\,x.$$

*(D) If the down payment parameter is higher than the user cost,* $\frac{R-\psi}{R} < \theta \leq 1$*,* $c(x)$ *is a concave function of* $x$ *and* $k(x)$ *is a convex function of* $x$*.*

Proposition 2 describes the shapes of the policy functions when the agent is liquidity constrained. These depend on the market credit conditions characterized by the relationship between the down payment parameter and the user cost. Figure 2 illustrates the propositions and depicts the policy rules for the four cases.

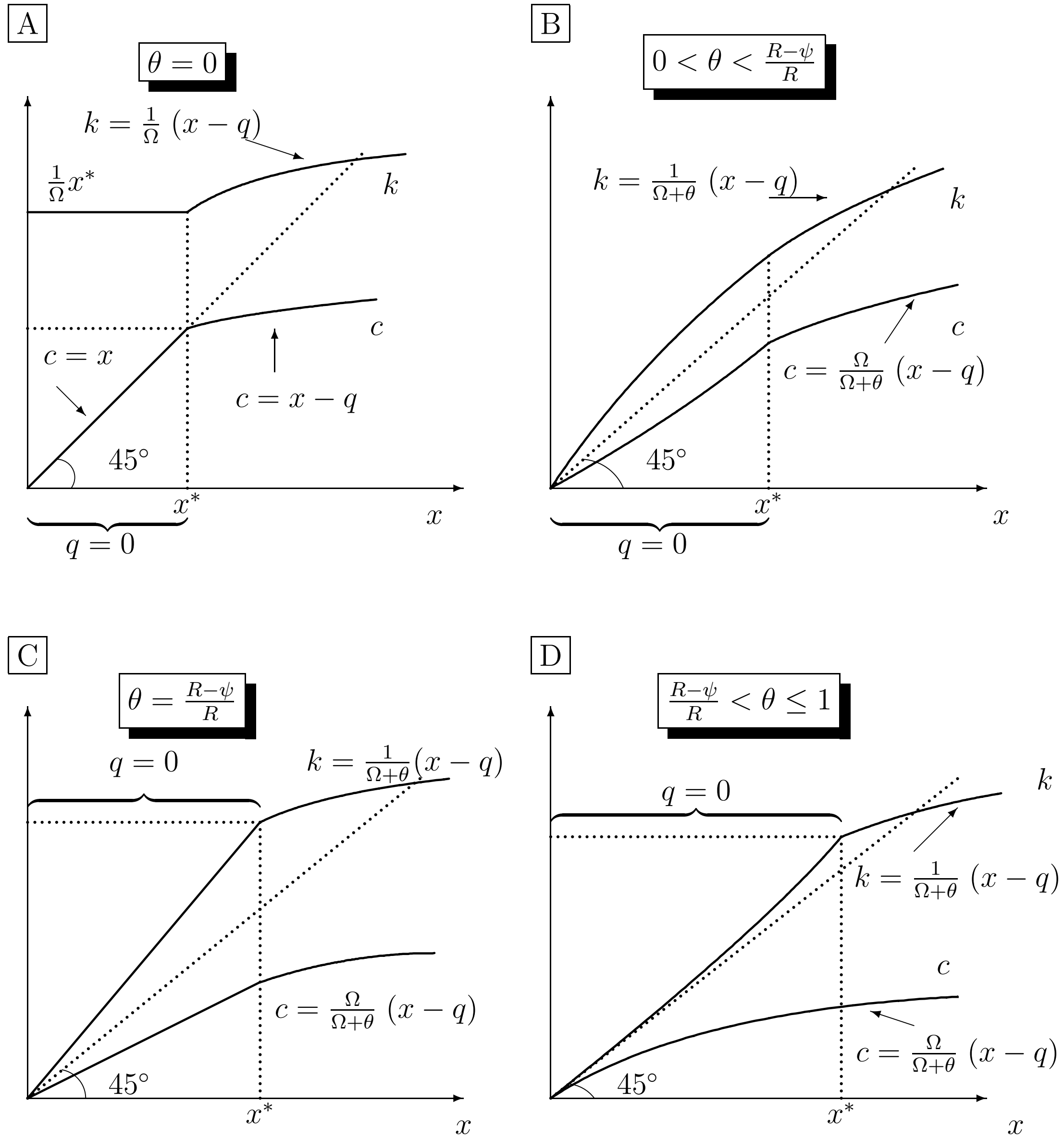

Figure 2: Policy Functions. Durable and Nondurable as a Function of Cash-on-Hand

Simple inspection of the functions tells us that we should not expect a big difference in the behavior of agents facing different down payment parameters when agents are not liquidity constrained. However, the differences should be obvious when they are constrained.

Note that the policy function for the durable good becomes "flatter" as the down payment requirement gets lower. As $\theta$ decreases, it gets easier to finance the durable, and current cash-on-hand is not so decisive. We move from a convex curve, to a straight line, to a concave curve, to a flat curve. The opposite is true for nondurable consumption. Note that for a level of normalized cash-on-hand such that the agent is constrained in all regimes, the marginal propensity to consume the nondurable good out of cash-on-hand is higher the lower is $\theta$ and it is exactly one when $\theta = 0$. These observations suggest higher nondurable volatility and lower durable volatility for regimes with lower down payments but do not prove it. Note that as $\theta$ gets higher, agents are more likely to be liquidity constrained for a given level of $x$. Moreover, some levels of $x$ may be more likely in one regime than in others. In other words, for an accurate study of the implications of different down payments regimes on consumption patterns, we must turn to simulations.

# 4 Implications of Lower Down Payments on Consumption Behavior

In order to fully understand the effect that down payment requirements have on consumption behavior, we calculate several consumption statistics for both nondurable and durable goods under different down payment regimes (different $\theta$s) through numerical simulation. We perform simulations for an individual and for an aggregate of individuals to compare the microeconomic and macroeconomic implications of the model.

## 4.1 Calibration of Parameters

We use an annual horizon as most microeconomic evidence from the parameters that we must calibrate come from studies of annual data. The following baseline parameter values were used. We set the relative risk aversion coefficient $\rho = 2$. The rate of time preference is set equal to 0.05, $\beta = 1/1.05$, and the net real interest rate equal to 0.02, $R = 1.02$. The income parameters are as follows: $\mu_G = 0.02$, $\mu_N = \mu_V = 0$, $\sigma_G = 0.025$, $\sigma_N = 0.05$, and $\sigma_V = 0.07$. All values are similar to Ludvigson & Michaelides (2001), whose results we would like to compare to ours.[16]

$\psi$, the depreciation factor, is set to 0.915, implying an annual depreciation rate of 8.5%. We obtained this number by combining data from the *National Income and Product*

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[16] In fact, these parameter values correspond to one of their worst performing scenarios for the standard buffer stock model of savings. All results are robust to small variations of the parameters. Results are not included here for brevity.

*Accounts* (NIPA) and the *Fixed Assets and Consumer Durable Goods* (FACD) of the Bureau of Economic Analysis. We assume that the U.S. is in a steady state and calculate the real, average ratio of investment on durables, $I$, and the durable stock, $K$. This ratio determines the depreciation rate:[17]

$$1 - \psi = \frac{I/Y}{K/Y}.$$

We are interpreting durables in a comprehensive manner as the sum of housing and all durable goods. Note that if $R = 1.02$ and $\psi = 0.915$, the user cost of the durable, $(R-\psi)/R = 0.1029 \simeq 0.1$. A down payment of approximately 10% will be the benchmark case in which the liquidity constraint does not impose distortions in the allocation of income across goods for a given period.

We need to calibrate one last parameter, $\varphi$, the preference parameter in the utility function. We proceed as follows. First we calibrate the ratio of nondurable consumption to the durable stock $(C/K)$ using NIPA and FACD, and find it to be 0.36.[18] Moreover, we know that when an agent is not constrained:

$$\frac{C}{K} = \left(\frac{R-\psi}{R}\right)^{\frac{1}{\rho}} \varphi^{-\frac{1}{\rho}}.$$

Given the values of $\rho$, $R$, $\psi$ and $C/K$, we obtained $\varphi = 0.795$. We use the former value for our individual consumption simulations and let the $C/K$ ratio vary accordingly to illustrate the effects of $\theta$ on the ratio. For the aggregate consumption simulations, $\varphi$ is adjusted to keep the ratio $C/K$ constant and equal to 0.36 under the different down payment regimes.

It is important to take into account that certain aspects of the collateralized constraint that we consider in this paper deviate from financial contracts written in reality. Here, the borrowing rate is not a function of $\theta$. However, mortgages with low down payments, say 5%, usually come with higher effective interest rates through the purchase of compulsory private mortgage insurance. On the other hand, financial contracts do not allow debtors to adjust net worth as easily as it is possible in this model. We chose this specification for tractability, and at the same time, to gain some intuition about the role of

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[17] In the steady state, $\Delta K = I - (1-\psi)K = 0$ which implies $1 - \psi = I/K = (I/Y)/(K/Y)$. Output, $Y$, in our case is GDP minus services in housing, since in our model these services enter through the stock of durable.

[18] $C$ is defined as the sum of nondurable consumption plus services minus housing and $K$ as the sum of the private residential stock plus the stock of consumer durables. Both amounts are normalized by GDP minus housing. We chose to keep shoes and clothing within the nondurable category since in this framework it does not seem appropriate to model them as durables. The results of the paper do not change significantly if these are ignored or treated as durables.

different financing possibilities for durables on the behavior of consumption, in particular nondurable consumption. It is not obvious then what the proper value of $\theta$ should be for our calibration. $\theta = 0.2$ is the average down payment for the U.S. housing market for the last few decades.[19] However, given the fact that we are modelling all durables (not only houses), we consider a range for $\theta$ between [0.2, 0.4] to be acceptable for the U.S. economy, and consider the complete range of the down payment parameter space in our simulations, $\theta \in [0, 1]$. Note that down payment parameters below the user cost represent very favorable credit conditions for the purchase of durable goods: the long term cost of buying the durable (the user cost) is higher than the short term cost of the durable (the down payment).[20]

The parameters above are used to calculate optimal policy rules for nondurable and durable consumption. For the simulations, we generate labor income shocks from the assumed distribution of idiosyncratic and aggregate earnings for 220 periods. Given the computed optimal policy rules and an initial distribution of financial assets, we can calculate nondurable and durable consumption (for that number of periods). When calculating our statistics, we ignore the first 20 periods of each simulation to insulate the results from the influence of initial conditions.[21] For an agent, the statistics reported are individual averages for the considered period for 20,000 samples. For the aggregate, we first compute cross section averages over 2,000 agents and then averages for the considered period for 100 samples.

## 4.2 Individual Consumption Results

Table 1 reports the results for the consumption of one individual whose income follows the process described in Section 2 under different down payment regimes. We generate individual time series using the method outlined above. We report the average and the standard deviation for durable and nondurable consumption growth, the nondurable to durable consumption ratio, the proportion of time that the agent is liquidity constrained, as well as excess smoothness and excess sensitivity coefficients. The excess sensitivity coefficient is the OLS coefficient from a regression of consumption growth on lagged income growth and a constant. The excess smoothness coefficient is the ratio of the standard

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[19] See for example Chiuri & Jappeli (2001).

[20] We do not believe down payment parameters below the user cost to be very plausible but we will present results over the entire down payment parameter space for completeness.

[21] Normalized assets and cash on hand converge to a stationary distribution very quickly, about 12 to 15 periods starting from zero assets.

deviation of consumption growth to that of labor income growth. Several results are worth pointing out.

Fist, note that when $\theta$ is equal to the user cost of the durable, the liquidity constraint does not affect the intratemporal allocation between the durable and nondurable. The agent spends fixed proportions of his cash-on-hand on both goods and consequently all reported statistics are identical for durables and nondurables.

Second, as the down payment increases, the average nondurable to durable ratio increases since buying the durable good is more costly in terms of current liquidity. Also, the agent is liquidity constrained more often, i.e. the agent's voluntary equity is zero a higher proportion of the time. Required equity increases with the down payment and agents choose not to carry much voluntary equity.

Third, nondurable consumption growth as well as its standard deviation decrease monotonically as the down payment requirement increases. The down payment may be burdensome, which implies lower nondurable consumption growth for higher down payments. On the other hand, the down payment provides required equity which may be used to protect nondurable consumption from negative income shocks when an agent runs out of voluntary equity, leading to lower volatility. Note than an agent can smooth nondurable consumption considerably for all possible down payments (the smoothness coefficient is well below 60% for down payments of 20% or higher). The smoothing is more efficient as the down payment parameter increases for the very same reason. The excess sensitivity coefficient is not significantly different from zero for down payments higher than the user cost and it is negative and significant for down payments lower than the user cost.

Finally, durable consumption growth first increases and then decreases with the down payment. However, the standard deviation of durable consumption growth is non-monotonic in $\theta$. In order to understand this non-monotonicity, it proves useful to decompose income shocks into transitory and permanent, which we will do next. Note that an agent can smooth durable consumption considerably as well.[22]

### 4.2.1 Transitory Shocks

In Table 1, we report the same statistics as before for an agent who receives idiosyncratic transitory shocks only.[23] The agent examined in this case is a buffer stock saver who holds

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[22] Recall that we are modelling durable consumption stocks, not durable consumption expenditure.

[23] We calculate the optimal policy functions for an agent who has uncertainty with regards to transitory idiosyncratic income shocks only. Idiosyncratic permanent shocks and common permanent shocks are set to their averages. All simulations are based on the new optimal policy functions calculated under these

Table 1. Model with no Adjustment Costs. Micro Results

| $\theta$ | 0.0 | 0.05 | $\sim 0.1^{\diamondsuit}$ | 0.2 | 0.3 | 0.4 | 0.5 | 1.0 |
|---|---|---|---|---|---|---|---|---|
| All Shocks (avg. $g_Y = 0.0263$, sd. $g_Y = 0.1107$) | | | | | | | | |
| avg. $g_C$ | 0.0249 | 0.0244 | 0.0238 | 0.0227 | 0.0221 | 0.0217 | 0.0215 | 0.0211 |
| sd. $g_C$ | 0.0970 | 0.0918 | 0.0842 | 0.0700 | 0.0592 | 0.0521 | 0.0478 | 0.0397 |
| Smoothness | 0.8771 | 0.8299 | 0.7606 | 0.6325 | 0.5352 | 0.4709 | 0.4326 | 0.3590 |
| | (0.0386) | (0.0334) | (0.0252) | (0.0160) | (0.0152) | (0.0163) | (0.0169) | (0.0173) |
| Sensitivity | -0.1712* | -0.1639* | -0.1401* | -0.0851 | -0.0380 | -0.0084 | 0.0041 | 0.0144 |
| avg. $g_K$ | 0.0222 | 0.0228 | 0.0238 | 0.0245 | 0.0239 | 0.0229 | 0.0223 | 0.0211 |
| sd. $g_K$ | 0.0622 | 0.0717 | 0.0842 | 0.0928 | 0.0859 | 0.0730 | 0.0631 | 0.0392 |
| Smoothness | 0.5622 | 0.6478 | 0.7606 | 0.8381 | 0.7759 | 0.6601 | 0.5700 | 0.3547 |
| | (0.0245) | (0.0217) | (0.0252) | (0.0213) | (0.0168) | (0.0167) | (0.0164) | (0.0146) |
| Sensitivity | -0.0529 | -0.1038* | -0.1401* | -0.1355* | -0.0773 | -0.0177 | 0.0095 | 0.0336 |
| $C/K$ | 0.3503 | 0.3550 | 0.3600 | 0.3695 | 0.3799 | 0.3905 | 0.4008 | 0.4489 |
| Constrained | 40.94% | 42.80% | 52.09% | 72.64% | 91.12% | 98.54% | 99.82% | 100.00% |
| Idiosyncratic Permanent Income Shocks Only (avg. $g_Y = 0.0213$, sd. $g_Y = 0.0483$) | | | | | | | | |
| avg. $g_C$ | 0.0221 | 0.0216 | 0.0213 | 0.0210 | 0.0209 | 0.0208 | 0.0208 | 0.0207 |
| sd. $g_C$ | 0.0643 | 0.0556 | 0.0483 | 0.0419 | 0.0390 | 0.0373 | 0.0361 | 0.0330 |
| Smoothness | 1.3303 | 1.1509 | 1.0000 | 0.8673 | 0.8064 | 0.7709 | 0.7468 | 0.6825 |
| | (0.0205) | (0.0107) | (0.0000) | (0.0080) | (0.0100) | (0.0105) | (0.0106) | (0.0102) |
| Sensitivity | -0.3018* | -0.1553* | -0.0051 | 0.1026 | 0.1178* | 0.1133* | 0.1056* | 0.0767 |
| avg. $g_K$ | 0.0213 | 0.0214 | 0.0213 | 0.0208 | 0.0206 | 0.0205 | 0.0204 | 0.0203 |
| sd. $g_K$ | 0.0483 | 0.0516 | 0.0483 | 0.0383 | 0.0316 | 0.0274 | 0.0246 | 0.0179 |
| Smoothness | 1.0000 | 1.0684 | 1.0000 | 0.7914 | 0.6532 | 0.5676 | 0.5096 | 0.3708 |
| | (0.0000) | (0.0050) | (0.0000) | (0.0132) | (0.0201) | (0.0238) | (0.0259) | (0.0298) |
| Sensitivity | -0.0051 | -0.0755 | -0.0051 | 0.1717* | 0.2356* | 0.2458* | 0.2391* | 0.1829* |
| C/K | 0.3490 | 0.3542 | 0.3600 | 0.3704 | 0.3812 | 0.3916 | 0.4019 | 0.4496 |
| LC | 99.46% | 99.75% | 100.00% | 100.00% | 100.00% | 100.00% | 100.00% | 100.00% |
| Idiosyncratic Transitory Income Shocks Only (avg. $g_Y = 0.0245$, sd. $g_Y = 0.0963$) | | | | | | | | |
| avg. $g_C$ | 0.0228 | 0.0226 | 0.0221 | 0.0213 | 0.0208 | 0.0205 | 0.0203 | 0.0201 |
| sd. $g_C$ | 0.0767 | 0.0735 | 0.0666 | 0.0520 | 0.0401 | 0.0312 | 0.0257 | 0.0147 |
| Smoothness | 0.7972 | 0.7638 | 0.6926 | 0.5401 | 0.4167 | 0.3248 | 0.2668 | 0.1525 |
| | (0.0386) | (0.0358) | (0.0269) | (0.0127) | (0.0077) | (0.0071) | (0.0065) | (0.0045) |
| Sensitivity | -0.2083* | -0.2130* | -0.2007* | -0.1432* | -0.0873* | -0.0469* | -0.0282 | -0.0057 |
| avg. $g_K$ | 0.0203 | 0.0210 | 0.0221 | 0.0233 | 0.0230 | 0.0221 | 0.0216 | 0.0205 |
| sd. $g_K$ | 0.0243 | 0.0448 | 0.0666 | 0.0825 | 0.0787 | 0.0663 | 0.0566 | 0.0336 |
| Smoothness | 0.2524 | 0.4655 | 0.6926 | 0.8568 | 0.8175 | 0.6887 | 0.5884 | 0.3497 |
| | (0.0101) | (0.0132) | (0.0269) | (0.0234) | (0.0148) | (0.0150) | (0.0143) | (0.0103) |
| Sensitivity | -0.0606* | -0.1338* | -0.2007* | -0.2283* | -0.1745* | -0.0994* | -0.0622 | -0.0130 |
| C/K | 0.3492 | 0.3545 | 0.3600 | 0.3700 | 0.3808 | 0.3917 | 0.4023 | 0.4514 |
| LC | 45.78% | 46.30% | 53.61% | 74.59% | 93.79% | 99.93% | 100.00% | 100.00% |

*Notes:* $g_C$ is the growth rate of nondurable consumption. $g_K$ is the growth rate of durable consumption. For all cases, $R = 1.02$, $\psi = 0.915$, $\varphi = 0.795$, $\beta = 1/1.05$, $\rho = 2$. The income statistics are as follows: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_V = 0$, $\sigma_N = 0.05$, and $\sigma_V = 0.07$. Results shown are averages for 200 periods taken over $20,000$ samples. Standard deviation of the simulation in parentheses.

$\diamondsuit$ down payment equal to user cost.

* significant at 5% level or better.

wealth (despite his impatience) to smooth out temporary shocks to income. As $\theta$ increases, an agent's voluntary equity decreases and his required equity increases. At the extremes,

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assumptions.

for $\theta = 0$ the agent holds no required equity and for $\theta = 1$ the agent holds no voluntary equity. In all our simulations, as $\theta$ increases, required equity increases are higher than the decreases in voluntary equity. Therefore, total wealth holdings increase with $\theta$. More wealth translates into better consumption smoothing for the nondurable.

The agent's behavior is depicted in Figure 3, which plots transitory shocks, normalized nondurable consumption and total and voluntary equity for two different down payment regimes, 10% and 30%. Note how normalized total (voluntary) equity is much higher (lower) for the 30% down payment regime. As a result, normalized nondurable consumption is smoother.

What about the durable? If an agent faces a transitory negative income shock, after voluntary equity runs out, required equity holdings are liquidated and used to smooth nondurable consumption. As $\theta$ increases, the agent is constrained a greater portion of the time resulting in more occasions where durable consumption is reduced to turn required wealth into nondurable consumption. One may expect then to observe less smoothing of the durable as $\theta$ increases. However, this is not the case for the entire range of $\theta$. Eventually durable consumption gets smoother as the down payment increases as well. This is because for high values of $\theta$, the amount of forced savings is so high that the efficiency of transforming required wealth holdings into nondurable consumption increases and requires a less dramatic reduction in durable consumption. Since the wealth requirement is proportional to the durable stock, the doubling of the wealth requirement implies halving the amount of durable consumption reduction necessary to yield a given amount of liquid resources. This effect allows the agent to liquidate more wealth without as great a reduction in durable consumption. As a consequence, volatility of durable consumption starts to fall again as the down payment requirement becomes higher.

Figure 3 also depicts normalized nondurable and durable consumption for three different down payment regimes, 10%, 30% and 100%. Note that although higher down payments are associated with smoother nondurable consumption, the relationship with durable consumption is less obvious.

### 4.2.2 Permanent Shocks

We repeat the experiment now for an agent who receives idiosyncratic permanent shocks only.[24] Results are reported in Table 1.

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[24] We calculate the optimal policy functions for an agent who has uncertainty with regards to permanent idiosyncratic income shocks only. Idiosyncratic transitory shocks and common permanent shocks are set to their averages. All simulations are based on the new optimal policy functions calculated under these

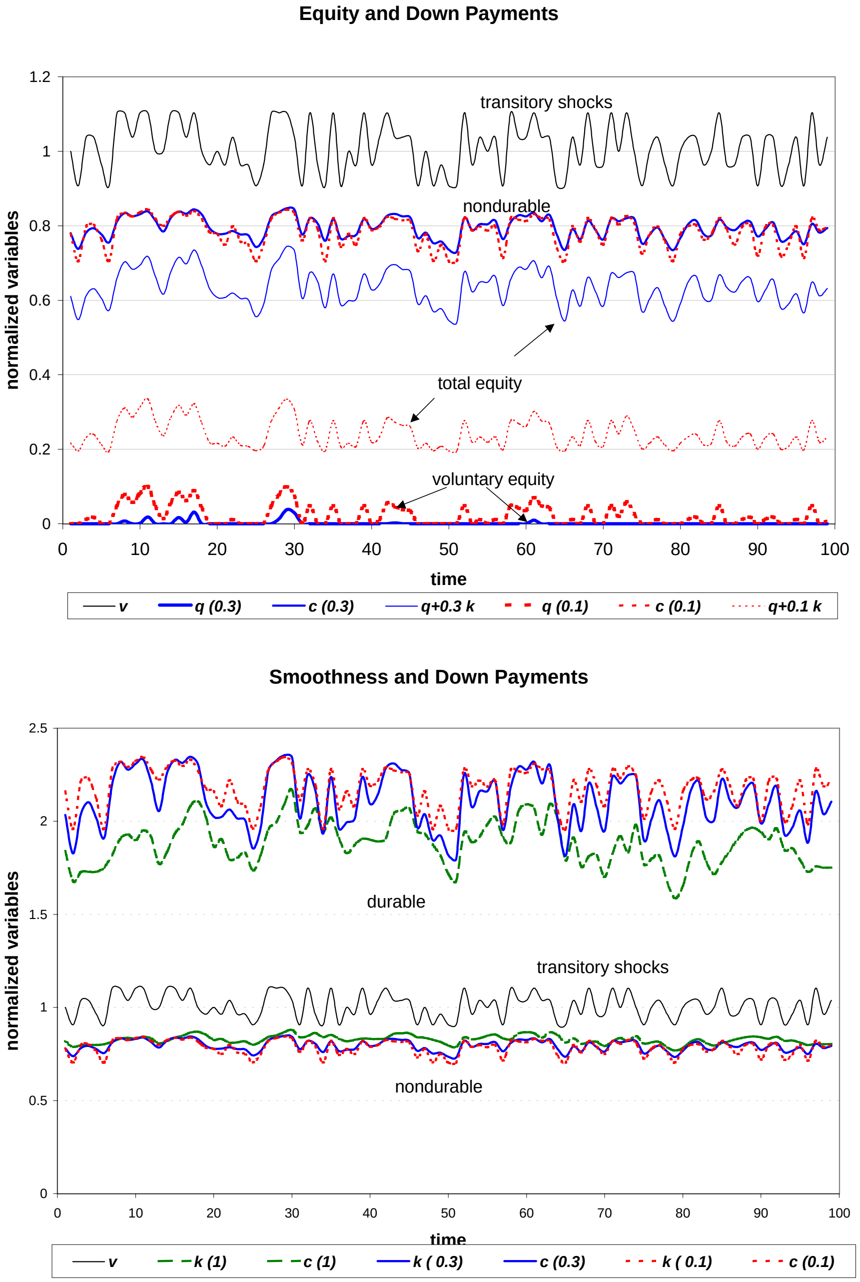

Figure 3: Transitory Shocks, Smoothness and Equity

First, note that nondurable consumption growth and its standard deviation decrease with the down payment parameters as in the previous section. For down payment parameters higher than the user cost, nondurable consumption is less volatile than income and *vice versa*. The relative smoothness coefficient is equal to 1 for a down payment equal to the user cost.[25] The same pattern is observed for the durable when the down payment is greater than the user cost.

In this case, the relationship between volatility and the down payment can be explained as the agent spreading out the accumulation or liquidation of required wealth holdings in response to an income shock. For example, if there is an above-average permanent income shock, the agent's new equilibrium level of durable consumption increases. This higher level of durable consumption, however, requires more wealth holdings in the form of down payment. Instead of increasing required wealth immediately to its equilibrium level, the agent accumulates the new down payment requirement over time so that nondurable consumption does not suffer temporarily. As $\theta$ increases, the burden imposed by the down payment increases resulting in more smoothing of the required down payment.

The degree of smoothing is controlled by whether the down payment requirement is greater than, less than, or equal to the durable good's user cost, $(R-\psi)/R$. The user cost can be thought of as the durable good's long-run price in terms of the agent's intertemporal budget constraint. When the liquidity constraint binds, the short-run cost of the durable is effectively $\theta$, the cost of a unit of durable consumption in terms of foregone nondurable consumption. When these two prices equate and the agent is constrained, the wealth requirement imposes a cost (in terms of foregone nondurable consumption) that is the same as the user cost. This sends a signal to the agent to adjust fully to the new equilibrium level of durable consumption. When the short-run cost of the durable is greater than its long-run cost (when $\theta > (R-\psi)/R$), the agent gradually adjusts to the new equilibrium level of durable consumption over several periods. A full adjustment would impose too great a sacrifice of nondurable consumption relative to the unconstrained situation. When the short-run cost is lower than the long-run cost, the agent has favorable credit conditions and overshoots the equilibrium levels of consumption and then gradually asymptotes back to the new equilibrium consumption level.

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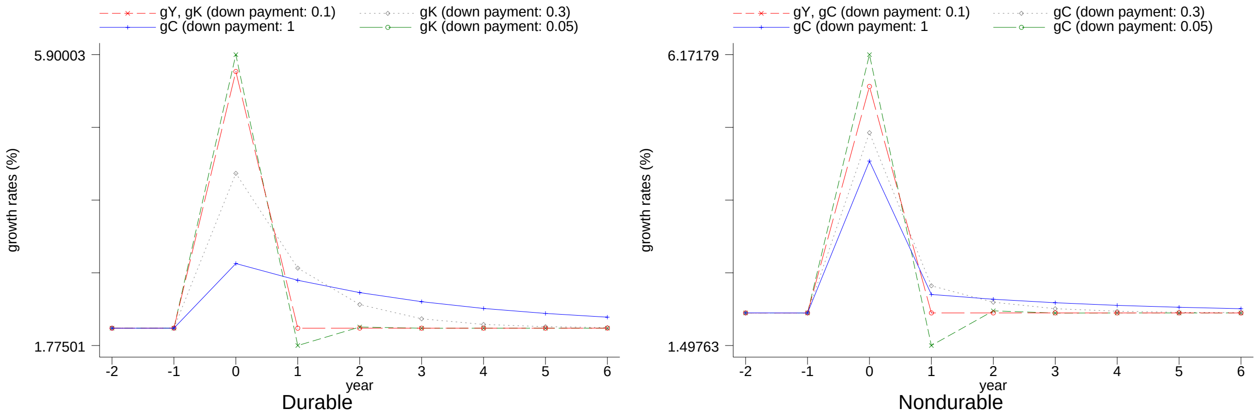
assumptions.

[25] When there are only permanent shocks to income, the agent is always liquidity constrained in our model. By Proposition 2, we know that when the down payment parameter equals the user cost, durable and nondurable consumption are a linear function of cash-on-hand. Moreover, it is easy to show that in this case cash-on-hand equals income. Therefore the growth rate of consumption (durable and nondurable) equals the growth rate of income.

Figure 4 illustrates these results. Consider an agent who has been receiving average income shocks for a series of periods. In period 0, he receives an above average permanent income shocks. After this period, all shocks go back to average. We consider four different down payment regimes, 5% and 10% – which, for our calibration, corresponds to the value in which user cost and down payment equate – as well as 30% and 100%.



Durable

Nondurable

Figure 4: ADJUSTMENT TO PERMANENT SHOCKS

For a down payment equal to the user cost, consumption – both durable and nondurable – adjusts fully in the period of the shock. For the 5% down payment – which is lower than the user cost – consumption overshoots, while for the 30% and 100% down payment cases, consumption does not fully adjust in the period of the shock. The full adjustment process is the longest for the highest down payment parameter.

Finally, note that the slow adjustment when the down payment is above the user cost produces both excess smoothness and excess sensitivity for the durable and then nondurable, as we can see in Table 1. The overshooting pattern observed when the down payment is below the user cost generates excess volatility and negative excess sensitivity for the nondurable and excess volatility an no excess sensitivity for the durable.

### 4.2.3 Summary

Two main points can be drawn from the previous discussion. On one hand, down payments are a form of *forced savings*. This means that consumers facing higher down payments have more wealth which they can use to smooth nondurable consumption when facing *transitory* income shocks. On the other hand, down payments, when high, may be a burden. Agents may choose not to adjust consumption levels immediately when facing *permanent* income shocks and prefer to spread out the accumulation or liquidation of required wealth holdings in response to them. Consequently, we should observe an agent living in an economy with higher down payments, facing both transitory and permanent income shocks, to have a smoother nondurable consumption growth than an agent facing the same income process in an economy with lower down payment requirements. It is important to realize that the agent living in the economy with lower down payments has more volatile nondurable consumption growth, however he experiences higher average nondurable consumption growth.

With regards to the durable, one must take into account that it provides consumption services and at the same time it is an asset. When an agent has used all his voluntary equity and needs to free resources for nondurable consumption, he must change his durable stock to free equity. As the down payment increases, an agent has less voluntary equity and adjusts his durable stock more often, which suggests higher durable consumption volatility for higher down payments. However, if the agent has a lot of required equity (because of high down payments), he may not need to change his durable stock significantly to free resources. This explains the non-monotonicity of durable volatility. Again, average consumption growth moves in sync with its volatility.

As Table 1 shows, excess smoothness is not associated with significant excess sensitivity for an agent – facing the income process with transitory and permanent shocks – in this model. Finally, one should realize that an agent who faces lower down payments is better off because he is facing a less restricted liquidity constraint. Consequently, he can finance a higher proportion of his durable purchases. The first three panels in figure 5 depict durable and nondurable smoothness coefficients for different values of $\theta$ for the three cases discussed above.

## 4.3 Aggregate Consumption Results

We next explore the aggregate implications of the augmented buffer stock model of savings developed earlier. In particular, we would like to determine if the model can replicate macroeconomic stylized facts for nondurable and services as well as residential and durable stocks. First, we use U.S. aggregate data to illustrate the empirical puzzles for which we are trying to provide an explanation. We consider annual growth rates of real per capita series for the period 1959–1999. All flow series come from NIPA and all stock series from FADC. We consider the following series: labor income, consumption expenditure on nondurables and services (with and without housing services and expenditure on clothing and shoes) and

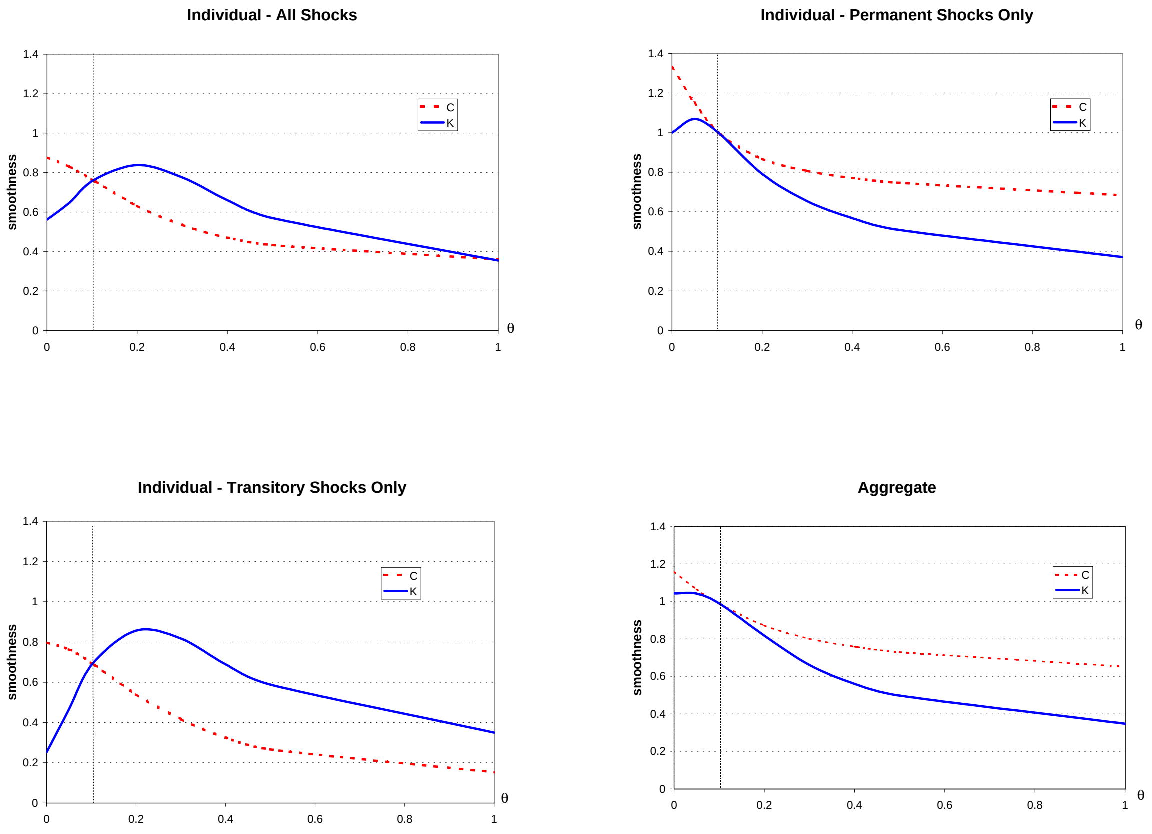


Figure 5: Smoothness as a Function of the Down Payment Parameter. No-Adjustment Cost Case.

the stock of durables and residences. Labor income is compiled from the NIPA components as wages and salaries plus other labor income minus personal contributions for social insurance minus taxes. Taxes are defined as the fraction of wage and salary income in total income, times personal tax and non tax payments. All series are per capita and deflated by their corresponding chain-type price deflator. Labor income is deflated by the Personal Consumption Expenditure chain-type price deflator. This data is summarized in Table 2.

In column 2 of Table 2, we present the empirical results. All expenditure series exhibit smoothness relative to labor income except the durable stock. All variables also exhibit excess sensitivity. The aggregates that best match our model are nondurable expenditure plus services minus housing services for $C$ and the residential stock plus the durable stock for $K$.[26] The statistics that referred to these series are the ones that we would like to explain with our model. In the data, $K$ is smoother than $C$ and its excess sensitivity coefficient is higher and more significant.[27]

As we mentioned before, in order to construct aggregate variables for our model, we generate idiosyncratic and aggregate labor income shocks from the assumed income distributions for 2,000 consumers during 200 periods. Given the calculated optimal consumption rules, we use those shocks to simulate nondurable and durable consumption for each consumer. Aggregate consumption in a given period is then calculated as the cross sectional average of individual consumption for all simulated consumers. Simulating more consumers did not change the results. Our results from this explicit aggregation are reported on Table 3. We also report results from a representative agent who receives the aggregate income process considered, $\Delta \ln Y_t = \ln G_t + 0.5\sigma_N^2$.

We can summarize the results of our simulations as follows. First, note that when the down payment parameter is equal to the user cost, the model does not deliver either excess smoothness or excess sensitivity and the results are similar to those for the standard buffer stock model of savings for the nondurable in Ludvigson & Michaelides (2001). In their Table 2, for our choice of parameters, they report an excess sensitivity coefficient of 0.989 and an excess smoothness value of 0.001.

Second, when the down payment is higher than the user cost, we obtain excess smoothness and sensitivity. As in the data, $K$ is smoother than $C$ and its excess sensitivity

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[26] In our specification, services from housing are derived from $K$, not from $C$.

[27] We acknowledge that referring to the excess sensitivity and excess smoothness of the durable is an abuse of terminology, as these terms refer to the empirical evidence of nondurable only. We report analogous coefficients for the durable to provide a first guideline of the fitness of our model to the durable empirical evidence as well.

coefficient is higher. Although for a reasonable range of $\theta$ – from 0.2 to 0.4 – our model cannot fully explain the excesses observed in the data, it certainly obtains much better results than the standard buffer stock model of savings.

Table 2. Stylized Facts of U.S. Data. 1959–1999

| | Excess Smoothness | Excess Sensitivity |
|---|---|---|
| $C$ | | |
| nondurables+services | 0.59 | 0.21 |
| | | (2.39) |
| nondurables+services-housing | 0.65 | 0.19 |
| | | (1.85) |
| nondurables+services-shoes and clothing | 0.55 | 0.23 |
| | | (2.75) |
| nondurables+services-housing-shoes and clothing | 0.60 | 0.20 |
| | | (2.16) |
| $K$ | | |
| consumer durables stock | 0.33 | 0.65 |
| | | (5.74) |
| residential stock | 0.97 | 0.19 |
| | | (4.19) |
| durables+residential stock | 0.40 | 0.26 |
| | | (5.10) |

*Notes: Excess Smoothness* is measured as the ratio of the standard deviation of the growth rate of a variable divided by the standard deviation of the growth rate of labor income. *Excess Sensitivity* refers to the OLS coefficient of a variable growth rate on the lagged labor income growth rate ($t$-statistic in parentheses). All flow series are from the *National Product and Income Accounts* and all stock series from *Fixed Assets and Consumer Durable Goods* published by the Bureau of Economic Analysis. They are all per capita and appropriately deflated. Labor income is compiled from the NIPA components as wages and salaries plus other labor income minus personal contributions for social insurance minus taxes. Taxes are defined as the fraction of wage and salary income in total income, times personal tax and non tax payments. This measure is also per capita and is deflated by the PCE chain-type price deflator.

Moreover, excess smoothness increases monotonically with increases in the down payment. This fact should not come as a surprise. In the aggregate, idiosyncratic shocks tend to cancel each other out and what remains is the influence from the common shock, which is permanent. As we have seen, agents choose not to adjust consumption levels immediately when facing permanent income shocks and prefer to spread out the accumulation or liquidation of required wealth holdings in response to them. The higher the down payment, the more burdensome this is and the longer the adjustment process lasts. This slow adjustment carries over to the aggregate.

On the other hand, excess sensitivity increases first and then decreases as the down payment parameter increases. The best way to explain the intuition behind this non-monotonicity is through the following controlled experiment. Consider an agent who has been receiving average income shocks for a series of periods. In period 0, he receives an above average permanent income shock. After this period, all shocks go back to average. Figure 6 depicts the paths for the growth rates of nondurable and durable consumption for three different down payments, 15%, 30% and 100%.

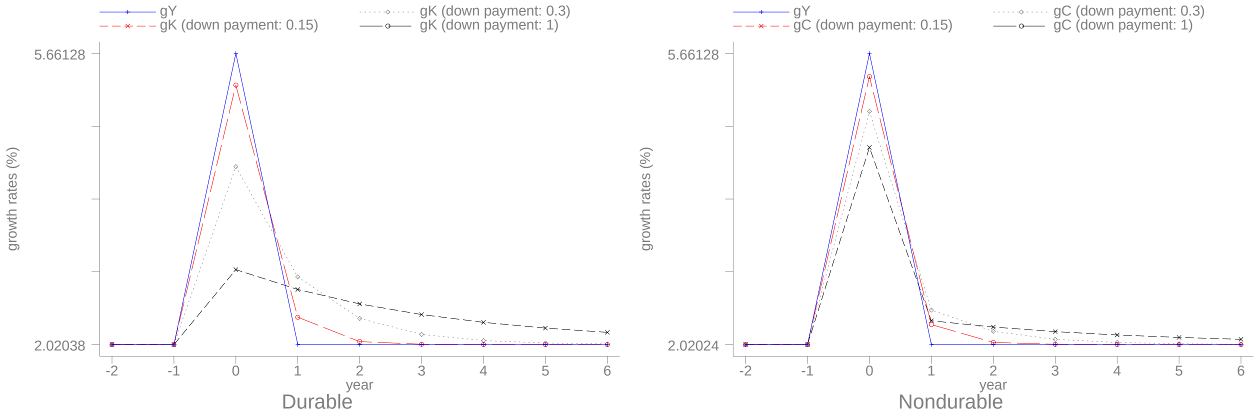

Figure 6: Smoothness and Sensitivity as a Function of the Down Payment. No-Adjustment Cost Case.

Durable and nondurable consumption do not fully react to the change in income in the period of the shock. The higher the down payment, the smaller the initial adjustment and the longer the total adjustment period. The excess sensitivity coefficient calculated in our simulations measures the reaction of current consumption changes to changes in income in the previous period only. Consider period 1 in the graph. Consumption growth is the lowest for the lowest down payment, the highest for the intermediate down payment and intermediate for the higher down payment. This corresponds to the pattern of excess sensitivity observed in our simulations. It can be seen that if the initial consumption adjustment is very small (compared to a lower down payment regime), the consumption growth rate in the next period may very well lay below that of a lower down payment regime, resulting in a lower excess sensitivity coefficient, in spite of the total adjustment period being longer.[28]

Finally, note that we obtain similar results from our explicit aggregation and from the exercise that considers a representative agent who receives the aggregate income process. This is due to the fact in the aggregate all idiosyncratic shocks get cancelled.

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[28] This would suggest that a proper measure of excess sensitivity should consider changes in income growth from previous periods as well.

Table 3. Model with no Adjustment Costs. Aggregate Results

| $\theta$ | 0 | 0.05 | ~0.1$^\diamondsuit$ | 0.2 | 0.3 | 0.4 | 0.5 | 1 |
|---|---|---|---|---|---|---|---|---|
| Explicit Aggregation | | | | | | | | |
| avg $g_C$ | 0.0218 | 0.0217 | 0.0217 | 0.0216 | 0.0216 | 0.0216 | 0.0216 | 0.0215 |
| sd $g_C$ | 0.0284 | 0.0262 | 0.0241 | 0.0214 | 0.0196 | 0.0186 | 0.0179 | 0.0160 |
| Smoothness | 1.1600 | 1.0650 | 0.9849 | 0.8721 | 0.8010 | 0.7582 | 0.7303 | 0.6520 |
| | (0.0095) | (0.0060) | (0.0042) | (0.0083) | (0.0116) | (0.0129) | (0.0132) | (0.0130) |
| Sensitivity | -0.1176 | -0.0604 | 0.0012 | 0.0786 | 0.1104* | 0.1129* | 0.1059* | 0.0757 |
| | (0.0822) | (0.0760) | (0.0701) | (0.0619) | (0.0565) | (0.0534) | (0.0514) | (0.0461) |
| avg $g_K$ | 0.0217 | 0.0217 | 0.0217 | 0.0216 | 0.0215 | 0.0215 | 0.0215 | 0.0214 |
| sd $g_K$ | 0.0256 | 0.0255 | 0.0241 | 0.0200 | 0.0162 | 0.0137 | 0.0122 | 0.0085 |
| Smoothness | 1.0426 | 1.0409 | 0.9849 | 0.8169 | 0.6605 | 0.5603 | 0.4972 | 0.3477 |
| | (0.0061) | (0.0053) | (0.0042) | (0.0111) | (0.0204) | (0.0259) | (0.0287) | (0.0331) |
| Sensitivity | -0.0341 | -0.0388 | 0.0012 | 0.1213* | 0.2117* | 0.2385* | 0.2323* | 0.1688* |
| | (0.0742) | (0.0741) | (0.0701) | (0.0575) | (0.0446) | (0.0361) | (0.0313) | (0.0217) |
| $\varphi$ | 0.740 | 0.770 | 0.795 | 0.840 | 0.890 | 0.940 | 0.890 | 1.250 |
| Representative Agent who receives Aggregate Income | | | | | | | | |
| avg. $g_C$ | 0.0207 | 0.0206 | 0.0205 | 0.0204 | 0.0204 | 0.0204 | 0.0204 | 0.0204 |
| sd. $g_C$ | 0.0321 | 0.0278 | 0.0242 | 0.0209 | 0.0195 | 0.0186 | 0.0180 | 0.0165 |
| Smoothness | 1.3306 | 1.1508 | 1.0000 | 0.8674 | 0.8065 | 0.7710 | 0.7468 | 0.6824 |
| | (0.0204) | (0.0812) | (0.0000) | (0.0079) | (0.0099) | (0.0105) | (0.0106) | (0.0101) |
| sensitivity | -0.3025* | -0.1555* | -0.0051 | 0.1026 | 0.1180* | 0.1136* | 0.1060* | 0.0772 |
| | (0.0923) | (0.0812) | (0.0712) | (0.0614) | (0.0568) | (0.0543) | (0.0527) | (0.0522) |
| avg. $g_K$ | 0.0205 | 0.0206 | 0.0205 | 0.0204 | 0.0204 | 0.0203 | 0.0203 | 0.0203 |
| sd. $g_K$ | 0.0242 | 0.0258 | 0.0242 | 0.0191 | 0.0158 | 0.0137 | 0.0123 | 0.0090 |
| Smoothness | 1.0000 | 1.0687 | 1.0000 | 0.7917 | 0.6538 | 0.5683 | 0.5103 | 0.3717 |
| | (0.0000) | (0.0050) | (0.0000) | (0.0131) | (0.0200) | (0.0236) | (0.0258) | (0.0296) |
| Sensitivity | -0.0051 | -0.0758 | -0.0051 | 0.1716* | 0.2356* | 0.2460* | 0.2394* | 0.1836* |
| | (0.0712) | (0.0759) | (0.0712) | (0.0551) | (0.0434) | (0.0365) | (0.0321) | (0.0230) |

*Notes:* $g_C$ is the growth rate of nondurable aggregate consumption. $g_K$ is the growth rate of durable aggregate consumption. Aggregate consumption is constructed by taking average consumption over $2,000$ individuals. For all cases, $R = 1.02$, $\psi = 0.915$, $\beta = 1/1.05$, $\rho = 2$. The income statistics are as follows: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_V = 0$, $\sigma_N = 0.05$, and $\sigma_V = 0.07$. Results shown are averages for 200 periods taken over 100 samples. The average income growth and the standard deviation of income growth are 0.0217 and 0.0245, respectively. $\varphi$ varies with $\theta$ to keep the ratio constant $C/K$=0.36.

$\diamondsuit$ denotes the value of the down payment equal to the user cost, $\theta = (R-\psi)/R$, which for our particular choice of parameters is roughly 0.1.

* significant at 5% level or better.

# 5 The Model with Adjustment Costs

In the first part of this paper we have worked under a very strong assumption about the durables market: there were no costs associated with additional changes in the durable stock. This does not reflect reality very well, since one of the well observed characteristics of the durables market is its illiquidity. People do not move every day or change the car they drive often, and when they do, they encounter several costs (search costs, taxes and transaction costs). In this section, we extend the model to allow for adjustment costs in the durable market. We consider non convex costs of adjustment as proposed by Grossman & Laroque (1990). These generate $(S, s)$ adjustment behavior. In most periods, consumers do not adjust their durable stock but when they do, they usually make substantial changes. Technically, this is not a trivial modification.

Under the new specification, financial assets, $A_t$, evolve according to:

$$A_t = RA_{t-1} - (K_t - \psi\ (1 - d\phi)K_{t-1}) + Y_t - C_t\ . \tag{18}$$

where $d$ is a dummy variable which takes on the value of zero when there is no investment and a value of one when investment is non-zero.

$$d = \begin{cases} 0, & \text{when } K_t - \psi K_{t-1} = 0 \\ 1, & \text{when } K_t - \psi K_{t-1} \neq 0\ . \end{cases} \tag{19}$$

$\phi$ is the adjustment cost parameter. Note that once the agent has decided to adjust his durable holdings, the adjustment cost is fixed from the agent's perspective. The cost is proportional to the "inherited" level of the durable stock, $\psi K_{t-1}$. This adjustment cost can also be thought of as the cost incurred upon sale of the agent's prior holdings of the durable stock, where $\phi$ represents a proportional loss in the selling price. As mentioned in Eberly (1994), this loss in price can be attributable to any type of cost incurred upon sale, such as the payment of taxes, a sales commission, or an imperfection in the resale market for the durable.[29]

Unfortunately, we cannot use the numerical technique utilized in the first part of the paper to solve the model with adjustment costs. When there are adjustment costs, the solution to the agent's problem will take the form of an $(S, s)$ rule. The value function will have a kink at $s$ and Euler equation iteration, a method based on taking derivatives, cannot be used. We resort to a different numerical dynamic programming technique, a *Finite State Approximation* method. In order to apply this technique, we must use an alternative reformulation of the model. We cannot reduce the dimensionality of the problem to just one state variable and we must work with two. ($A_t$, $K_t$) is the natural choice pair, but it proves very inconvenient.[30] Instead, we work in terms of ($Q_t$,$K_t$), voluntary equity and

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[29] If $\phi$ is to be interpreted as the proportional loss in the selling price of the durable, (19) implicitly assumes that there is no incremental adjustment (i.e. the agent must sell his entire existing stock upon adjustment). See Lam (1989) for an analysis of the aggregate implications of the time series properties of durable expenditure when the irreversibility of incremental adjustment of the durable is due to resale market imperfections.

[30] The problem is that with $K_t > 0$ (from preferences) and $A_t \geq -(1 - \theta)K_t$, the feasible region in the

the durable stock.

The evolution of voluntary equity is given by:

$$\begin{aligned} Q_t &\equiv A_t + (1-\theta)K_t \\ &= RA_{t-1} - (K_t - \psi(1-d\phi)K_{t-1}) + Y_t - C_t + (1-\theta)K_t \\ &\quad + R(1-\theta)K_{t-1} - R(1-\theta)K_{t-1} \\ &= RQ_{t-1} + [\psi(1-d\phi) - R(1-\theta)]K_{t-1} - \theta K_t + Y_t - C_t \ . \end{aligned}$$

The liquidity constraint becomes $Q_t \geq 0, \ \forall t$. In order to deal with the nonstationarity of income, we normalize all variables by permanent income. We can then use the homogeneity of degree $(1-\rho)$ property of the utility function to write the Bellman equation of the model.

$$\begin{aligned} & V(q_{t-1}, k_{t-1}) = \\ & \quad \beta \, \mathrm{E}_{t-1} \Big\{ (G_t N_t)^{1-\rho} \max_{q_t, k_t; q_t \geq 0} U \Big[ (G_t N_t)^{-1} \Big\{ Rq_{t-1} + [\psi(1-d\phi) - R(1-\theta)]k_{t-1} \Big\} \\ & \quad\quad -\theta k_t + V_t - q_t, k_t \Big] + V(q_t, k_t) \Big\} \ . \end{aligned}$$

It is easy to show that the first order conditions of the problem when $\phi = 0$ are identical to the ones presented in section 2. The solution technique is described in Appendix A.2. Briefly, it consists of specifying and solving a finite-state problem that approximates the continuous one presented above. Note that under this formulation, the control variables $(q_t, \ k_t)$ are also next period's states. These continuous variables can be approximated by finite discrete sets. Moreover, the specification allows for a straightforward incorporation of the liquidity constraint and the adjustment costs in the durable market. Then, the discretely approximated problem is solved using value function iteration combined with an acceleration technique, *modified policy function iteration*, as explained in the appendix.

Simulation results are presented in Table 4 and Figure 7, where we assume a 5% adjustment cost. The solution of this model contains several non-trivial features which distinguish it from the no-adjustment cost model.

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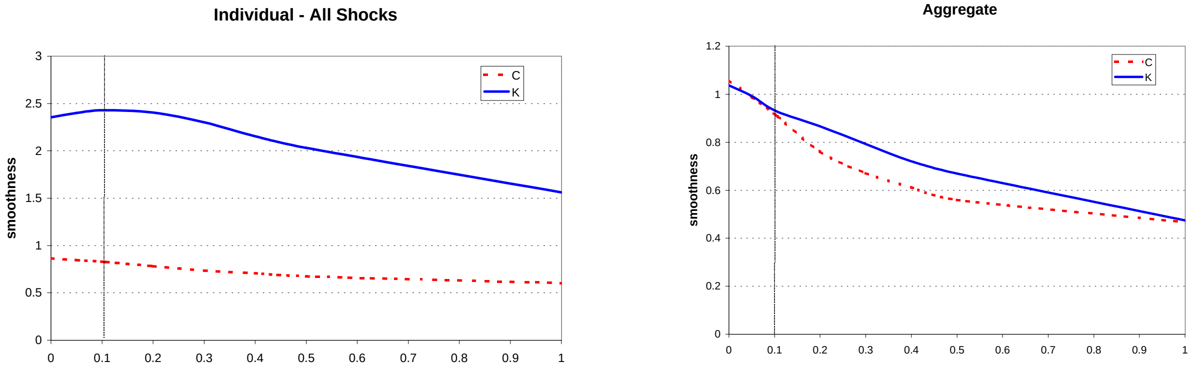
$(A_t, K_t)$ space is not rectangular.

Table 4: Model with Adjustment Costs. Results

| $\theta$ | 0 | 0.05 | $\sim 0.1^{\diamondsuit}$ | 0.2 | 0.3 | 0.4 | 0.5 | 1 |
|---|---|---|---|---|---|---|---|---|
| Individual Results (avg. $g_Y = 0.0263$, sd. $g_Y = 0.1107$) | | | | | | | | |
| avg. $g_C$ | 0.0247 | 0.0245 | 0.0243 | 0.0239 | 0.0234 | 0.0232 | 0.0230 | 0.0225 |
| sd. $g_C$ | 0.0957 | 0.0936 | 0.0915 | 0.0865 | 0.0812 | 0.0782 | 0.0745 | 0.0668 |
| Smoothness | 0.8646 | 0.8461 | 0.8275 | 0.7821 | 0.7347 | 0.7079 | 0.6747 | 0.6049 |
| | (0.0426) | (0.0422) | (0.0431) | (0.0439) | (0.0438) | (0.0447) | (0.0436) | (0.0386) |
| Sensitivity | -0.1610* | -0.1525* | -0.1346* | -0.0829 | -0.0334 | 0.0127 | 0.0365 | 0.0688 |
| avg. $g_K$ | 0.0460 | 0.0467 | 0.0472 | 0.0468 | 0.0450 | 0.0425 | 0.0405 | 0.0333 |
| sd. $g_K$ | 0.2600 | 0.2651 | 0.2683 | 0.2657 | 0.2542 | 0.2379 | 0.2243 | 0.1724 |
| Smoothness | 2.3543 | 2.4001 | 2.4293 | 2.4058 | 2.3014 | 2.1538 | 2.0308 | 1.5609 |
| | (0.1290) | (0.1347) | (0.1383) | (0.1352) | (0.1272) | (0.1176) | (0.1093) | (0.0825) |
| Sensitivity | -0.0360 | 0.0104 | 0.0527 | 0.1205 | 0.1219 | 0.1080 | 0.0849 | 0.0944 |
| Aggregate Results (explicit aggregation) (avg. $g_Y = 0.0217$, sd. $g_Y = 0.0245$) | | | | | | | | |
| avg. $g_C$ | 0.0217 | 0.0217 | 0.0216 | 0.0216 | 0.0215 | 0.0215 | 0.0215 | 0.0215 |
| sd. $g_C$ | 0.0260 | 0.0242 | 0.0216 | 0.0186 | 0.0164 | 0.0150 | 0.0137 | 0.0114 |
| Smoothness | 1.0587 | 0.9872 | 0.9134 | 0.7605 | 0.6707 | 0.6108 | 0.5595 | 0.4666 |
| | (0.0071) | (0.0067) | (0.0074) | (0.0160) | (0.0214) | (0.0248) | (0.0277) | (0.0273) |
| Sensitivity | -0.0628 | -0.0295 | 0.0075 | 0.1024* | 0.1542* | 0.1800* | 0.1978* | 0.1604* |
| avg. $g_K$ | 0.0217 | 0.0217 | 0.0228 | 0.0216 | 0.0216 | 0.0215 | 0.0215 | 0.0215 |
| sd. $g_K$ | 0.0254 | 0.0243 | 0.0558 | 0.0212 | 0.0194 | 0.0176 | 0.0164 | 0.0116 |
| Smoothness | 1.0367 | 0.9921 | 0.9298 | 0.8662 | 0.7926 | 0.7201 | 0.6689 | 0.4741 |
| | (0.0259) | (0.0276) | (0.0302) | (0.0366) | (0.0364) | (0.0364) | (0.0310) | (0.0302) |
| Sensitivity | 0.1021 | 0.1530* | 0.2265* | 0.3262* | 0.3866* | 0.4099* | 0.3727* | 0.2931* |
| $\varphi$ | 0.9 | 0.95 | 1.0 | 1.1 | 1.2 | 1.25 | 1.3 | 1.6 |

*Notes:* $g_C$ is the growth rate of nondurable aggregate consumption. $g_K$ is the growth rate of durable aggregate consumption. Aggregate consumption is constructed by taking average consumption over $2,000$ individuals. For all cases, $R = 1.02$, $\psi = 0.915$, $\beta = 1/1.05$, $\rho = 2$ and $\phi = 0.05$. The income statistics are as follows: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_V = 0$, $\sigma_N = 0.05$, and $\sigma_V = 0.07$. Individual results shown are averages for 200 periods taken over 20,000 samples. Aggregate results are averages for 200 periods taken over 100 samples. $\varphi$ varies with $\theta$ to keep the ratio constant $C/K$=0.36 for the aggregate simulation. $\varphi = 0.795$ for the individual level simulations.
$\diamondsuit$ denotes the value of the down payment equal to the user cost, $\theta = (R - \psi)/R$, which for our particular choice of parameters is roughly 0.1.
* significant at 5% level or better.

First, to minimize the cost of adjustment, the agent changes the durable stock infrequently. Adjusting the durable to its no-adjustment equilibrium level each period would be costly. Thus, the agent follows a $(S, s)$ rule for adjustment of the durable stock. Second, during the agent's holding period for the durable good, required wealth necessarily falls as the durable depreciates. The agent, however, saves as he anticipates the down payment requirement needed to change his durable stock during the adjustment period. The agent also engages in additional, minimal savings to pay for the adjustment cost. This is done very close to the period of adjustment. Third, the $(S, s)$ rule used by agents implies that, unlike the no-adjustment cost case, consumption of an individual who receives aggregate income may not exhibit the same properties as aggregate consumption. One reason is that agents have different times until they adjust the durable stock. Thus, one should expect to see a greater persistence of the effect income has on aggregate consumption as individuals reach their triggers at different times. This is in fact what we find.


Individual - All Shocks

Aggregate

Figure 7: Smoothness As a Function of the Down Payment Parameter. Adjustment Cost Case.

At the individual level, we observe that the durable stock is now more volatile than income and nondurable consumption – while smoother than income for all $\theta$ values – and is slightly more volatile than in the case with no adjustment costs.[31] For the aggregate, the volatility of consumption growth for both goods monotonically increases as $\theta$ falls, just as before. Recall, that in the no adjustment case, if $\theta > (R - \psi)/R$, an above-average permanent shock would lead to an accumulation of required wealth over several periods. This would lead to a gradual adjustment of consumption to its equilibrium level. Under adjustment costs, the same phenomenon is occurring. The only difference is the $(S, s)$ pattern of adjustment towards the new equilibrium level. As a result, nondurable consumption turns out to be smoother with adjustment costs. In fact, for down payments between 30 and 40%, the model can reproduce the actual excess sensitivity and smoothness observed in the data. The durable, however, is more volatile than what we observe in the data under this new specification. In summary, the model with adjustment costs can reproduce the excess sensitivity and excess smoothness observed in the data for the nondurable.

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[31] We leave analysis of the micro case for future study and show it here for completeness only.

# 6 Conclusions

This paper considers a buffer stock model of savings where agents consume both durable and non-durable goods. Agents face a minimum net worth constraint incorporating a down payment which captures the idea that borrowing to finance durables is often less costly than borrowing to finance nondurables, since the durable can serve as collateral. Moreover, we consider variations of the model with and without adjustment costs in the durable market. We show that this liquidity constraint can alter the allocation of resources between the durable and nondurable and has implications for the volatility of the two goods. Indeed, only when the down payment requirement *exactly equals* the user cost of the durable should we study durables and nondurables separately.

We find that for an individual, nondurable consumption growth unambiguously becomes more volatile when the required down payment for purchases of durables in an economy is lowered. Moreover, this result is preserved by aggregation. For the durable, the results are slightly more complex, and while some non-monotonicities exist at the individual level with respect to volatility, durable consumption is also more volatile for a lower down payment at the aggregate level. For an individual, this result is explained in part by the fact that higher down payment requirements translate into higher wealth holdings to deal with negative transitory income shocks – since down payments act as a form of forced savings – and in part by the fact that consumers choose to gradually adjust their consumption when facing permanent income shocks to spread out the cost of the down payment. For the aggregate, it is only this last effect that survives. Moreover, this gradualism or sluggish response of consumption to permanent income shocks generates robust excess smoothness and excess sensitivity in an explicit aggregation of the model for plausible parameter values. The model with adjustment costs strongly matches the empirical evidence for nondurable consumption. At the individual level, we obtain excess smoothness, but not necessarily robust excess sensitivity.[32]

This model also predicts that countries with higher down payments (in relation to the appropriate user cost) should exhibit higher smoothness and higher excess sensitivity of nondurable consumption, which is consistent with the empirical evidence in Japelli & Pagano (1989), but which has yet to be explained by the literature (Ludvigson & Michaelides (2001) and Michaelides (2001)).[33]

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[32] This may help explain why part of the empirical evidence with household data does not find robust excess sensitivity, while the aggregate evidence is less controversial.

[33] Recall that in our simulations, excess sensitivity increases with excess smoothness first and although it may decrease eventually, this is the result of considering only one lagged income changes when calculating

Finally, we should stress that lower down payment requirements imply that households voluntarily lower wealth holdings (because of their impatience) and voluntarily accept the cost of greater volatility. In this model, consumers should be better-off in an economy with lower down payments.

Several factors not included in the model may further refine the results of this paper and deserve attention for future research. First, as noted in the Japanese economy, some home buyers simply give up saving for housing down payments as the cost is too prohibitive. An interesting extension to the current model would be to allow for this phenomenon which would require the explicit presence of a rental market. Second, the model could be generalized to include stochastic durable prices. Finally, we would like to consider the model in a general equilibrium framework.

# A Appendix. Numerical Procedures

## A.1 Euler Equation Iteration

### Derivation of key equations (12) and (13)

Using our particular utility specification, we can write equations (7) and (8), the Euler-Lagrange necessary conditions, as follows:

$$-C_t^{-\rho} + \beta R \,\mathrm{E}_t \left\{ \left[ C_{t+1}(X_{t+1}) \right]^{-\rho} \right\} + \lambda_t = 0 \tag{7'}$$

$$K_t^{-\rho} - \beta(R-\psi) \,\mathrm{E}_t \left\{ \left[ C_{t+1}(X_{t+1}) \right]^{-\rho} \right\} - \theta\lambda_t = 0 \tag{8'}$$

As in Carroll (1997), we rewrite the equations in ratio terms, by taking advantage of the homogeneity of degree $\rho$ of marginal utility and dividing all variables by permanent income:

$$-c_t^{-\rho} + \beta R \,\mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left[ c_{t+1}(x_{t+1}) \right]^{-\rho} \right\} + \lambda_t P_t^{\rho} = 0, \tag{7''}$$

$$k_t^{-\rho} - \beta(R-\psi) \,\mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left[ c_{t+1}(x_{t+1}) \right]^{-\rho} \right\} - \theta\lambda_t P_t^{\rho} = 0, \tag{8''}$$

---

the excess sensitivity coefficients. Japelli & Pagano (1989) include additional lags.

where

$$x_{t+1} \equiv \frac{X_{t+1}}{P_{t+1}} = (G_{t+1}N_{t+1})^{-1}[R(x_t - c_t) + (\psi - R)k_t] + V_{t+1},$$

$c_t \equiv C_t/P_t$ and $k_t \equiv K_t/P_t$.

When the agent is not liquidity constrained, $\lambda_t = 0$. Moreover, as we know from the intratemporal condition, $c_t/k_t = \Omega$. Using these facts and the definition of $x_{t+1}$ above, equation ($7''$) can be written as:

$$\beta R \,\mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left( c_{t+1} \left[ (G_{t+1}N_{t+1})^{-1} \left( R(x_t - c_t) + \left( \frac{\psi - R}{\Omega} \right) c_t \right) + V_{t+1} \right] \right)^{-\rho} \right\}$$
$$-c_t^{-\rho} = 0, \tag{12}$$

When the agent is constrained, $x_t = c_t + \theta k_t$. Also, we can solve for $\lambda_t P_t^{\rho}$ in equation ($7''$). Substituting into equation($8''$), we can write:

$$\beta[\psi - R(1-\theta)] \,\mathrm{E}_t \left\{ (G_{t+1}N_{t+1})^{-\rho} \left( c_{t+1} \left[ (G_{t+1}N_{t+1})^{-1} [\psi - R(1-\theta)] \frac{x_t - c_t}{\theta} + V_{t+1} \right] \right)^{-\rho} \right\}$$
$$-c_t^{-\rho} + \varphi \left( \frac{x_t - c_t}{\theta} \right)^{-\rho} = 0. \tag{13}$$

**About the Technique**

With the two equations above, we are ready to find the optimal rule for normalized nondurable consumption as a function of the unique state variable, normalized cash-on-hand, $x$. We will denote the optimal rule as $c(x)$.

Euler equation iteration requires assuming a finite horizon, $T$ and recursively solving backwards from the last period of life. To apply the method successfully, we need to (i) evaluate the expectation, (ii) select an appropriate terminal condition and (iii) find a criterion to check if the agent is liquidity constrained.

In order to evaluate the expectation, we avoid numerical integration by replacing the continuous $G_t$, $N_t$ and $V_t$ processes by 5-point discrete approximations as suggested by Tauchen (1986). With regards to the terminal condition, we will assume that, as in Deaton (1992), the value of total assets is zero at time $T$, $a_T + k_T = 0$. Then $c_T(x) = x$ (the agent spends all his cash-on-hand on the nondurable).

In period $T-1$, for a given value of $x$, we can numerically compute the value $c_{T-1}$

that satisfies the appropriate equation: the first one if the agent is not constrained and the second one if he is. We do so for a grid of values of $x$ and numerically approximate the optimal consumption rule $c_{T-1}(x)$ through interpolation between the points of the $x$ grid (we use cubic spline interpolation). Once we have $c_{T-1}(x)$, the same grid of $x$ values is used to compute $c_{T-2}(x)$. With $c_{T-2}(x)$, $c_{T-3}(x)$ is computed, and so on.

Note that there is an easy way to check if the agent is liquidity constrained for a given value of $x$. At each time iteration, find $x_t^*$, the exact value of cash-on-hand for which the liquidity constraint just binds. This can be done by noticing that at this point, $x_t^* = c_t(x_t^*) + \theta k_t(x_t^*)$ and $c_t(x_t^*) = \Omega k_t(x_t^*)$. This implies that $c_t(x_t^*) = \Omega(\Omega+\theta)^{-1}x_t^*$. Then we can just solve equation (12) for $x_t^*$. For all $x \leq x_t^*$ the agent is constrained, and *vice versa.*

Once we have the optimal policy function for the nondurable, $c(x)$, the optimal policy function for the durable, $k(x)$, can be backed out by using the intratemporal relationship between the two goods.

$$k(x) = \begin{cases} \theta^{-1}[x - c(x)], & x \leq x^*, \\ \\ \Omega^{-1}c(x), & x \geq x^*, \end{cases}$$

## A.2 Finite State Approximation

The technique consists of specifying a finite-state problem that approximates the continuous one that we are trying to solve. We replace the continuous state variables, $k$ and $q$ with the finite sets, $\mathcal{K} = \{k_1, \ldots, k_{N_k}\}$ and $\mathcal{Q} = \{q_1, \ldots, q_{N_q}\}$. Note that the problem has been conveniently formulated in such a way that control variables today are next period's states. The liquidity constraint is implemented by setting $q_1 = 0$ and $q_i > 0, \forall q_i \in \mathcal{Q},\ i > 1$.

To deal with adjustment cost, we set

$$d = \begin{cases} 0, & |k_t - (G_tN_t)^{-1}\psi k_{t-1}| \leq \kappa \\ 1, & |k_t - (G_tN_t)^{-1}\psi k_{t-1}| > \kappa \end{cases}$$

where $\kappa = (k_n - k_l)/(N_k - 1)$.

Note, that the precision of our solution increases as $\kappa$ falls. This "work around" solution may have some economic significance. It may be possible for the agent to make small changes to his durable stock, such as repairs, which do not require significant adjustment costs. If this is the case, the numerical formulation described here would be most appropriate.

As with the previous technique, all components of the income process are discretized. $N_G$ points for $G_t$, $N_N$ for $N_t$ and $N_V$ for $V_t$. We then use value function iteration, which is sped up with an acceleration technique, *modified policy function iteration with $S$ states.*[34] Briefly,

1. Choose an initial guess $V^0$. Let $V^\ell = V^0$.

2. Calculate $U^{\ell+1} = \mathcal{U}V^\ell$. For each $(q_i, k_j)$, the mapping $\mathcal{U}$ is defined as

$$\begin{aligned} U^{\ell+1}_{i,j,m,n,o} \quad = \quad & \max_{q^+,k^+;q^+\geq 0} U\Big[(G_m N_n)^{-1}\Big\{Rq_i + [\psi(1-d\phi) - R(1-\theta)]k_j\Big\} \\ & -\theta k^+ + V_o - q^+, k^+\Big] + V^\ell(q^+, k^+) \equiv \mathcal{U}V^\ell_{i,j} \end{aligned}$$

3. Let $W^0 = V^\ell$. For each $(q_i, k_j)$ and $s = 1, \ldots, S$, calculate:

$$\begin{aligned} & W^{s+1}(q_i, k_j) = \\ & \beta \frac{1}{(N_G N_N N_V)} \Big\{ \sum_{m=1}^{N_G} \sum_{n=1}^{N_N} \sum_{o=1}^{N_V} (G_m N_n)^{1-\rho} U\Big[(G_m N_n)^{-1}\Big\{Rq_i + [\psi(1-d\phi) - R(1-\theta)]k_j\Big\} \\ & -\theta U^{\ell+1}_k + V_o - U^{\ell+1}_e, U^{\ell+1}_k\Big] + W^s[U^{\ell+1}_q, U^{\ell+1}_k]\Big\} \end{aligned}$$

   Set $V^{\ell+1} = W^S$.

4. Iterate until convergence.

Note that the selection of appropriate bounds for the sets $\mathcal{K}$ and $\mathcal{Q}$ is key for the successful application of the technique. See Farr & Luengo-Prado (1999) for more information about this method.

For the construction of Table 3, we set $N_G = N_N = N_V = 5$; $N_k = 75$, $k_1 = 0.01$ and $k_{N_k} = 2.5$; $N_q = 35$, $q_1$=0 and $q_{N_q} = 0.3$ for $\theta \in [0, 0.4]$; $N_q = 45$, $q_1$=0 and $q_{N_q} = 0.4$ for $\theta = 0.5$; $N_q = 55$, $q_1$=0 and $q_{N_q} = 0.5$ for $\theta = 1$;

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[34] Every time a new policy function is computed, we calculate the value function that would result from using this policy function $S$ times. With the newly obtained value function, we compute the new optimal policy function and so on. See Judd (1997) for a general description of these procedures.

## A.3 Comparing Techniques

We solve the problem without adjustment costs for a few parameter values with both techniques. Results are shown in the table below. All statistics are fairly similar, which suggests that the finite state approximation technique produces reasonable results.

With regards to computational time, the procedures rank very differently. Convergence of a policy function using Euler Equation Iteration (EEI) takes between 25 and 30 iterations and a total time of 0.45 minutes. A typical simulation takes 3.5 minutes. Convergence of policy functions using Finite State Approximation (FSA) takes between 10 and 12 iterations and 75 minutes. A typical simulation takes 1.3 minutes.

EEI is a faster procedure but it cannot be used when introducing adjustment costs. Moreover, It must be tailored to deal with other specifications of utility functions and it implementation may not be feasible in all cases. FSA, a discrete approximation, is a more robust method. It can easily accommodate adjustment cost and different utility specifications.[35] We refer interested readers to Farr & Luengo-Prado (1999) for further details about these procedures.

# B Appendix. Proofs of Propositions

**Proof of Proposition 1**

We know from the intratemporal first order condition of the problem that the ratio of nondurable to durable consumption equals $\Omega$, regardless of the value of $\theta$. Moreover $x_t = c_t + \theta k_t + q_t$. Both conditions imply the results.

**Proof of Proposition 2**

*(A)* If $\theta = 0$, $x_t = c_t + q_t$. When the agent is liquidity constrained $q_t = 0$ and $c(x) = x$. On the other hand, $k(x)$ will not depend on $x$ at all. Note also that $k$ gives utility today but decreases cash-on-hand tomorrow ($\psi < R$). Therefore, there must be an optimal level of $k$ that the agent will choose, while $x$ is below $x^*$. Given that $k(x) = \Omega^{-1}\,(x - q)$, for $x \geq x^*$, a natural candidate for the optimal $k$ is $k^* = \Omega^{-1} x^*$:

*(B)* Since the agent is liquidity constrained, $x = c(x) + \theta k(x)$. Then, $1 = c'(x) + \theta k'(x)$, $0 = c''(x) + \theta k''(x)$, with $c'(x) > 0$ and $k'(x) > 0$, and $c(0) = k(0) = 0$. We also know

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[35] All programs are in C++. Calculations were performed on a Pentium III 700.

from the intratemporal condition that:

$$\frac{c(x^*)}{k(x^*)} = \Omega \quad \text{and} \quad \frac{c(x)}{k(x)} < \Omega, \quad \forall x < x^*.$$

The closer $x$ to $x^*$, the closer the agent is to not being liquidity constrained, and the lower the shadow price of the constraint should be. Therefore the $c$-$k$ ratio increases with $x$. The only way this can happen is with $c''(x) > 0$ and $k''(x) < 0$. Hence $c(x)$ must be convex and $k(x)$ concave, while the agent is liquidity constrained.

*(C)* We observe from the intratemporal first order condition that in this case, the agent is able to keep the $c$-$k$ ratio constant and equal to $\Omega$. Moreover $c(x) + \theta k(x) = x$. Both conditions imply the result.

*(D)* The proof is similar to *(B)*. In this circumstance, the $c$-$k$ ratio should be higher than $\Omega$ when the agent is liquidity constrained and decrease towards $\Omega$ as $x$ approaches $x^*$. The only way that can happen is with a concave $c(x)$ and a convex $k(x)$.

Table 5: Comparison of Techniques.Results for the No-Adjustment Cost Model.
(Individual Level)

| $\theta$ | | 0.05 | | ~0.1* | | 0.3 | |
|---|---|---|---|---|---|---|---|
| | | EEI | FSA | EEI | FSA | EEI | FSA |
| $c$ | avg. | 0.7771 | 0.7771 | 0.7795 | 0.7792 | 0.7886 | 0.7886 |
| | sd. | 0.0421 | 0.0425 | 0.0391 | 0.0392 | 0.0293 | 0.0295 |
| $g_C$ | avg. | 0.0244 | 0.0245 | 0.0238 | 0.0238 | 0.0221 | 0.0221 |
| | sd. | 0.0918 | 0.0926 | 0.0842 | 0.0840 | 0.0592 | 0.0595 |
| | smoothness | 0.8299 | 0.8373 | 0.7606 | 0.7591 | 0.5352 | 0.5377 |
| | | (0.0334) | (0.0336) | (0.0252) | (0.0268) | (0.0152) | (0.0158) |
| | sensitivity | -0.1639 | -0.1686 | -0.1401 | -0.1297 | -0.0380 | -0.0373 |
| | | (0.0518) | (0.0585) | (0.0533) | (0.0533) | (0.0380) | (0.0382) |
| $k$ | avg. | 2.1878 | 2.1883 | 2.1661 | 2.1684 | 2.0809 | 2.0810 |
| | sd. | 0.0777 | 0.0785 | 0.1086 | 0.1072 | 0.1492 | 0.1497 |
| $g_K$ | avg. | 0.0228 | 0.0229 | 0.0238 | 0.0237 | 0.0239 | 0.0239 |
| | sd. | 0.0717 | 0.0722 | 0.0842 | 0.0837 | 0.0859 | 0.0862 |
| | smoothness | 0.6478 | 0.6527 | 0.7606 | 0.7568 | 0.7759 | 0.7786 |
| | | (0.0217) | (0.0233) | (0.0252) | (0.0271) | (0.0168) | (0.0172) |
| | sensitivity | -0.1038 | -0.0988 | -0.1401 | -0.1353 | -0.0773 | -0.0778 |
| | | (0.0416) | (0.0460) | (0.0533) | (0.0531) | (0.0550) | (0.0552) |
| $C/K$ | | 0.3550 | 0.3598 | 0.3598 | 0.3593 | 0.3799 | 0.3799 |

*Notes:* $\theta$ is the down payment parameter. $k$ is normalized durable and $c$ is normalized nondurable. $g_K$ and $g_C$ are the growth rates of durable and nondurable consumption (in levels) respectively. $C/K$ is the ratio of nondurable to durable (in levels).

$*$ denotes the value of the down payment equal to the user cost, $\theta = (R - \psi)/R$, which for our particular choice of parameters is roughly 0.1. For all cases, $R = 1.02$, $\psi = 0.915$, $\beta = 1/1.05$, $\rho = 2$, $\varphi = 0.795$. The income statistics are as follows: $\mu_G = 0.02$, $\sigma_G = 0.025$, $\mu_N = \mu_V = 0$, $\sigma_N = 0.05$, and $\sigma_V = 0.07$.

EEI stands for Euler Equation Iteration. FSA stands for Finite State Approximation.

Results shown are averages for 200 periods taken over 100 samples.

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