Note: The Newsvendor Model with Endogenous Demand

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Abstract† (163 Words)

This paper considers a firm's price and inventory when it faces uncertain demand that depends on both price and inventory level. We extend the classic newsvendor model by assuming that expected utility maximizing consumers choose between visiting the firm and consuming an exogenous outside option. The outside option represents the utility the consumer forgoes when she chooses to visit the firm before knowing whether or not the product will be available. We investigate both the case in which the firm's price is exogenous and the case in which price is chosen optimally. The paper makes two contributions. First, we show that the firm holds more inventory, provides a higher fill rate, and earns higher profits when it internalizes the effect of its inventory on demand. Second, we show that in the endogenous price case the firm's two-dimensional decision problem can be reduced to two, sequential, single variable optimizations. As a result, the endogenous-price case is as easy to solve as the exogenous-price case.

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1. Introduction

Consumers prefer to purchase from stores that have fewer stockouts. Consistent with this claim, marketing studies have documented that consumers are more likely to switch stores after experiencing a stockout than after finding goods in stock. This behavior has motivated the use of shortage costs and service level constraints in the design of inventory policy. Sometimes shortage costs represent the cost to the firm of some emergency method of distribution, but more commonly shortage costs are artificially imposed penalties motivated by the loss of goodwill associated with stockouts. These instruments are valuable because they help the firm to capture a richer set of relationships between availability and profitability than is explained by the simple newsvendor model.[1] However little research has explored how firms should set stockout penalties optimally.[2] This paper presents a simple static rational choice model in which consumers have a preference for avoiding stockouts and suggests how firms might optimally modify their inventory policies.

Formally, the paper extends the classic newsvendor model by introducing an exogenous outside option for consumers. Consumers observe the firm's price and inventory, learn whether or not they value the good and the value of their outside option, and then decide between visiting the firm (and buying the good if it is available) and consuming their outside option. The greater is the firm's inventory the more likely is the consumer to obtain the good if she visits the firm and the less likely is the consumer to choose her outside option. So the firm's demand is a function of both its price and its

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[1] See Silver, Pyke, & Peterson (1998) and Nahmias (1997) for a discussion of shortage costs and the relationship between shortage costs and service level constraints.

[2] Oral, et al. (1972) surveyed distributors to try to measure the probability a customer would leave the firm after experiencing a stockout. The product of this probability and the present value of profits earned per consumer would be a good empirical approximation of the shortage cost. Chang & Niland (1967) also try to measure stockout costs

inventory level. The assumption that consumers observe the firm's price and inventory level and can compute the expected fill rate is strong. More often consumers can only observe the firm's inventory policy indirectly, through experience, word of mouth, or consumer research institutions. However this assumption enables us to give a simple static formulation of this complex dynamic problem.

We explore the inventory decision in the presence of the outside option first in a model with an exogenous price and then in a model where price is chosen optimally. The paper makes two contributions. First, we show that when the firm internalizes the effect of expected availability on its demand it maintains a higher inventory level, sets a higher fill rate and earns higher profits. Second, when price is endogenous we find that the firm's two-dimensional decision problem can be reduced to two, sequential, single variable optimizations. As a result, the endogenous-price case is as easy to solve as the exogenous-price case.

An example that fits our assumptions is the decision to go to the movies[3]. Each consumer chooses whether to stay home and watch television or to go out to the local cinema. It is reasonable to assume that consumers know how large the cinema is. If the cinema is more spacious, then a consumer will reason that the chance that the cinema sells out is smaller and the chance she will get a seat is larger, so the consumer will be more likely to go to the cinema in the first place.

Several papers have attempted to make the relationship between availability and profitability more explicit by considering dynamic models in which consumers future arrivals depend on their past experience with the firm. Research that considers the inventory decision when consumer demand is a function of past stockouts (but price is exogenous) includes Schwartz (1966 and 1970), Balcer (1980, 1983), Fergani (1976), and

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[3] The newsvendor model describes either inventories of perishable goods or capacity for a non-storable commodity, like transportation services or entertainment.

Robinson (1990).[4] More recently Hall and Porteus (1999) examine a dynamic duopoly model of firm inventory competition in which consumers switch suppliers at an exogenously specified rate after experiencing a stockout. However these papers have not shown that the behavior they hypothesize for consumers is optimal or considered how availability affects their demand. Indeed, it is not clear in these models why past experience should have *any* effect on rational consumers' decision-making since the models generally show that firms choose symmetric and stationary inventory policies. Much more promising is work by Gans (1999a and 1999b) who considers a dynamic model of a firm's inventory decisions and consumer choice in which consumers learn about uncertain service levels over time, but he also considers an exogenous price and places some restrictions on consumer behavior.

The paper is closely related to previous work on competition in availability. This literature began with a paper on perfectly competitive firms by Carlton (1978) and was extended to imperfect competition in Deneckere and Peck (1995) and Peters (1984). These authors show that firms choose higher inventory levels (or capacities) when they compete with each other in both price and availability. In this literature consumers visit only one firm, so the opportunity cost of learning whether or not the firm has the good is the forgone opportunity to visit a different firm. In contrast this paper makes the point that an opportunity cost alone is sufficient to cause a firm to increase its inventory levels, so it is the cost of search for availability that yields higher inventories in these models.

Lippman and McCardle (1997), and more recently Mahajan and van Ryzin (1999) also consider models of competition in inventories.[5] Both papers consider models with exogenous prices. Lippman and McCardle consider a one-shot model in which

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[4] In a single-period setting, Eilon (1965) presents a model in which mean demand depends on the probability of a stockout. Wang and Gerchak (2001) analyze a manufacturer selling to one or more retailers when demand is shelf-space-dependent.

[5] See also Parlar (1988), Karjalainen (1992), and surveys by Porteus (1990) and Cachon (1999).

consumers' decisions about which firm to visit are exogenous, so firms' inventories do not stimulate demand. Nevertheless inventories affect rivals because consumers are able to visit other firms if the first firm they visit stocks out. Mahajan and van Ryzin consider a rational sequential choice model of consumer behavior, however consumers can costlessly visit every firm and always do so in a predetermined order so again inventories do not stimulate demand.[6]

Section 2 presents the model. In Section 3 we present the exogenous price case and in Section 4 we present the endogenous price case. Section 5 offers concluding remarks.

2. The Model

Assumptions:

1) The Timing:
   a) The firm chooses its inventory (or capacity), *k,* and price, *p,* to maximize its expected profits.
   b) Each consumer observes
      i) whether their valuation for a single unit of the good is *V* or 0,
      ii) the value of their outside option *u,* and
      iii) the firm's choice of price *p* and inventory level *k,*

      but *does not* observe how many other consumers value the good.

[6] When aggregate demand is uncertain and customers can costlessly search among every firm, it is natural to expect some firms to specialize in high availability at high prices and others to specialize in low prices but low availability. In equilibrium low priced firms have higher capacity utilization rates and so earn the same profits as high priced firms. This equilibrium prediction was first characterized by Prescott (1976) (see also Bryant, 1980) and has also been studied by Eden (1990) and Dana (1999). Dana considers an extension of Prescott's model in which individual firms can offer the same good at many different prices (as happens in the airline industry for example).

c) Each consumer chooses either to visit the firm, in which case they receive utility $V - p$ if they are able to buy the good and utility 0 if they are not, or consume their outside option and receive utility *u,* whichever yields higher expected utility.

2) We assume individual consumers are very small, but that whether or not they value the good is correlated across consumers so the aggregate demand, which we denote *a,* is uncertain. Let the distribution of *a* be described by the continuously differentiable cumulative probability distribution $F(a)$ on the interval $[\underline{a},\bar{a}] \subset \Re^+$ with strictly positive probability density function $f(a)$. Let $\mu = \int_{\underline{a}}^{\bar{a}} af(a)da$ denote the expected value of a. We suppose that the distribution of *a* is common knowledge.

3) We suppose that all consumers who would like to buy the good have a common valuation of *V*. Let the distribution of *u* be described by the continuously differentiable cumulative probability distribution function $G(u)$ on the interval $[\underline{u},\bar{u}] \subset \Re^+$ with strictly positive probability density function $g(u)$. We assume that *u* is independently distributed across consumers and is independent of whether or not the consumer's valuation of the good is *V* or 0.

4) All consumers who choose to visit the firm have an equal probability of being able to purchase a unit of the good.

5) Technical Assumption: Let the ratio $R(u) \equiv g(u)/G(u)$ be such that $R(u)^2 > R'(u)$. This technical requirement guarantees uniqueness of the optimal policy. It is satisfied, for example, if $g(u)$ is a member of the broad class of PF_2 density functions (see Barlow and Proschan, 1965).

Assumption 4 implies that each consumer who chooses to visit the firm is sold the good with probability one when the firm's inventory exceeds total demand and with probability equal to the firm's inventory divided by total demand when total demand exceeds the firm's inventory.

Let $\hat{u}$ be the outside option of the marginal consumer – the one who is indifferent between consuming her outside option and visiting the firm. Since all consumers whose outside option is less than $\hat{u}$ visit the firm, $aG(\hat{u})$ denotes the firm's demand.

The firm's sales are $\min(k, aG(\hat{u}))$ so expected demand and expected sales are

$$E[Demand(\hat{u})] = \int_{\underline{a}}^{\bar{a}} aG(\hat{u}) f(a) da = \mu G(\hat{u}), \text{ and} \tag{1}$$

$$E[Sales(k,\hat{u})] = \int_{\underline{a}}^{\bar{a}} \min(k, aG(\hat{u})) f(a) da = \mu G(\hat{u}) - E[aG(\hat{u}) - k]^+, \tag{2}$$

where $\hat{u}$ depends on the firm's price and inventory level. The firm's ex ante fill rate is

$$S(k,\hat{u}) \equiv \frac{E[Sales(k,\hat{u})]}{E[Demand(\hat{u})]} = \frac{\int_{\underline{a}}^{\bar{a}} \min(k, aG(\hat{u})) f(a) da}{\mu G(\hat{u})} = \frac{\mu - E[a - k/G(\hat{u})]^+}{\mu}. \tag{3}$$

Since the fill rate depends on k and $\hat{u}$ only through the stocking factor $z = k/G(\hat{u})$ (see Petruzzi and Dada, 1999) the fill rate can be written as

$$s(z) = 1 - \frac{E[a - z]^+}{\mu}. \tag{4}$$

Since $E[a-z]^+ = \int_z^{\bar{a}} (a - z) f(a) da = \int_z^{\bar{a}} [1 - F(a)] da$, the fill rate is a strictly positive, continuous function of the stocking factor with derivative $s'(z) = (1 - F(z))/\mu > 0$.

Consumers care about the probability that they will receive the good if they visit the firm. Although ex post (given the realization of demand) this probability is equal to the firm's ex post fill rate, it also is possible to show that the ex ante expectation of the ex post fill rate is equal to the firm's ex ante fill rate[7]:

$$\begin{aligned} \varphi(k,\hat{u}) &\equiv E\left[\frac{Sales(k,\hat{u})}{Demand(\hat{u})}\right] = \int_{\underline{a}}^{\bar{a}} \frac{\min(k, aG(\hat{u}))}{aG(\hat{u})} \frac{af(a)}{\mu} da \\ &= \frac{\int_{\underline{a}}^{\bar{a}} \min(k, aG(\hat{u})) f(a) da}{G(\hat{u})\mu} = S(k,\hat{u}) = s(z) \end{aligned} \tag{5}$$

[7] Deneckere and Peck (1995) refer to the ex ante expectation of the ex post fill rate as the service rate.

Note that $\min(k, aG(\hat{u}))/(aG(\hat{u}))$ is the ex post fill rate given the realization *a,* and $af(a)/\mu$ is the consumer's ex ante conditional probability density of *a given that she values the good*. As in Deneckere and Peck (1995), because consumers are identical and condition only on their own demand (their only information about the realization of *a* is what they can infer from their own valuations), the ex ante expected probability that the consumers will receive the good is equal to the firm's ex ante fill rate.[8]

The firm's profits are

$$pE[Sales(k,\hat{u})] - ck \tag{6}$$

where the outside option of the marginal consumer, $\hat{u}$, is implicitly defined by

$$\hat{u} = \varphi(k,\hat{u})(V-p) = s(z)(V-p). \tag{7}$$

3. Exogenous Price

A. Myopic Policy

In the traditional newsvendor model price is exogenous and demand does not depend on the inventory policy. Suppose $p > c$. The traditional newsvendor problem is to find the k that maximizes (6) where $\hat{u}$ and p are both treated as exogenous variables, although in equilibrium $\hat{u}$ is determined by (7). Substituting $k = zG(\hat{u})$ into (6) yields a profit function that depends only on z and the exogenous variable $\hat{u}$:

$$\pi(z,\hat{u}) = pG(\hat{u})\left[\mu - \int_z^{\bar{a}}[1-F(a)]da\right] - czG(\hat{u}). \tag{8}$$

The first order condition is

$$\frac{\partial \pi(z,\hat{u})}{\partial z} = pG(\hat{u})[1-F(z)] - cG(\hat{u}) = 0 \tag{9}$$

which has a unique solution that is independent of $\hat{u}$,

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[8] If there were a public signal correlated with demand (such as the weather or a review in the newspaper) then the probability would depend on that signal as well and would not in general be equal to the fill rate.

$$z_m = F^{-1}\left(\frac{p-c}{p}\right), \tag{10}$$

where the subscript *m* is used to denote the optimal policy for the myopic firm. It follows immediately that $\pi(z_m) > 0$ for all $\hat{u} > \underline{u}$. The firm's optimal inventory level is

$$k_m = G(\hat{u}_m)F^{-1}\left(\frac{p-c}{p}\right), \tag{11}$$

where $\hat{u}_m$, determined by (7), is

$$\hat{u}_m = \hat{u}(z_m, p) = s\left(F^{-1}\left(\frac{p-c}{p}\right)\right)(V-p). \tag{12}$$

We make the natural assumption that $\hat{u}_m > \underline{u}$ so $\pi(z_m) > 0$.

B. The Optimal Policy

When the firm's price is exogenous and the firm is *not* myopic, that is when it understands the effect of inventory on demand, the firm will choose *z* to maximize (8) subject to (7). The first order condition is

$$\frac{\partial\pi(z,\hat{u})}{\partial z} + \frac{\partial\pi(z,\hat{u})}{\partial\hat{u}}\frac{d\hat{u}}{dz} = 0 \tag{13}$$

where

$$\frac{\partial\pi(z,\hat{u})}{\partial\hat{u}}\frac{d\hat{u}}{dz} = \frac{g(\hat{u})}{G(\hat{u})}\pi(z,\hat{u})s'(z)(V-p) > 0. \tag{14}$$

Let z_p denote any solution to (13) where the subscript *p* denotes the exogenous price case. From (9), (13), and (14) we know that $z_p > z_m$ since for all $z < z_m$ the first term of (13) is clearly non-negative and the second term is strictly positive. This implies $s(z_m) < s(z_p)$ and $\hat{u}_m < \hat{u}_p$ since $\hat{u}$ is increasing in *z*, and $k_m < k_p$ since

$k_m = G(\hat{u}_m)z_m < G(\hat{u}_p)z_p = k_p$. Thus the myopic firm holds less inventory, provides a lower service level, and earns less profits than an optimizing firm.[9]

The uniqueness of z_p is demonstrated in the Appendix.

4. Endogenous Price

A. The Myopic Policy

In the literature on the newsvendor model with an endogenous price (see Petruzzi and Dada, 1999, for a recent review) the firm chooses its price and inventory with the expectation that its demand depends on its price, *p*, but not on its inventory policy, *k*. In our model this problem is equivalent to finding the *p* and *k* that maximize (6) subject to

$$\hat{u} = \hat{s}(V - p). \tag{15}$$

In other words, the firm understands that its price, *p,* affects its demand by changing $\hat{u}$, but it treats $\hat{s}$ as exogenous. However in equilibrium $\hat{s}$ is equal to the firm's fill rate, $s(z)$.

We can rewrite the firm's constraint, (15), as

$$p = V - \frac{\hat{u}}{\hat{s}}. \tag{16}$$

Substituting the constraint and $k = zG(\hat{u})$ into (6) yields a profit function that depends only on *z* and $\hat{u}$:

$$\Pi(z,\hat{u}) = G(\hat{u})\left[V - \frac{\hat{u}}{\hat{s}}\right]\left[\mu - \int_z^{\bar{a}}[1 - F(a)]da\right] - czG(\hat{u}). \tag{17}$$

The first order conditions are

$$\frac{\partial \Pi(z,\hat{u})}{\partial z} = G(\hat{u})\left[\left[V - \frac{\hat{u}}{\hat{s}}\right][1 - F(z)] - c\right] = 0, \tag{18}$$

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[9] Even with a public signal of demand or heterogeneous valuations, the same managerial insights apply. In general, the myopic firm chooses a lower inventory level than the optimizing firm as long as its profits are increasing in demand and demand is increasing in the inventory level.

and

$$\frac{\partial \Pi(z,\hat{u})}{\partial \hat{u}} = \\ g(\hat{u})\left[\left[V - \frac{\hat{u}}{\hat{s}}\right]\left[\mu - \int_z^{\bar{a}}[1-F(a)]da\right] - cz\right] - \frac{G(\hat{u})}{\hat{s}}\left[\mu - \int_z^{\bar{a}}[1-F(a)]da\right] = 0. \tag{19}$$

In equilibrium $\hat{s} = s(z)$ from (4), which, substituted into (19), yields

$$g(\hat{u})\left[V\left[\mu - \int_z^{\bar{a}}[1-F(a)]da\right] - \hat{u}\mu - cz\right] - \mu G(\hat{u}) = 0. \tag{20}$$

Let z_m^* and $\hat{u}_m^*$ denote the optimal policy for the myopic firm. Then z_m^* and $\hat{u}_m^*$ are given by (18) and (19), and $\hat{s} = s(z_m^*)$.

B. The Optimal Policy

When the firm chooses its price and inventory optimally, understanding that both effect demand, the firm's problem is to choose the *p* and *k* that maximize (6) subject to (7). Substituting $p = V - \hat{u}/s(z)$, from (7), and $s(z)$, from (4), and $k = zG(\hat{u})$ into (6), yields a profit function that depends only on *z* and $\hat{u}$:

$$\Pi(z,\hat{u}) = G(\hat{u})V\left[\mu - \int_z^{\bar{a}}[1-F(a)]da\right] - G(\hat{u})\mu\hat{u} - czG(\hat{u}). \tag{21}$$

The two first order conditions are

$$\frac{\partial \Pi(z,\hat{u})}{\partial z} = G(\hat{u})\left[V[1-F(z)] - c\right] = 0, \tag{22}$$

and

$$\frac{\partial \Pi(z,\hat{u})}{\partial \hat{u}} = g(\hat{u})\left[V\left[\mu - \int_z^{\bar{a}}[1-F(a)]da\right] - \mu\hat{u} - cz\right] - \mu G(\hat{u}) = 0. \tag{23}$$

Let z^* and $\hat{u}^*$ denote a solution to these two conditions. From (22),

$$z^* = F^{-1}\left(\frac{V-c}{V}\right), \tag{24}$$

so z^* is unique and independent of $\hat{u}^*$. From (23)

$$\frac{d\Pi(z^*,\hat{u})}{d\hat{u}} = -\mu G(\hat{u}) + R(\hat{u})\Pi(z^*,\hat{u}) \tag{25}$$

where, recall, $R(\hat{u}) = g(\hat{u})/G(\hat{u})$. Since

$$\Pi(z^*,\hat{u})\Big|_{\frac{d\Pi(z^*,\hat{u})}{d\hat{u}}=0} = \frac{\mu G(\hat{u})}{R(\hat{u})} \tag{26}$$

it follows that

$$\left.\frac{d^2\Pi(z^*,\hat{u})}{d\hat{u}^2}\right|_{\frac{d\Pi(z^*,\hat{u})}{d\hat{u}}=0} = -\mu g(\hat{u}) + R'(\hat{u})\frac{\mu G(\hat{u})}{R(\hat{u})} = -\frac{\mu G(\hat{u})}{R(\hat{u})}\left[R(\hat{u})^2 - R'(\hat{u})\right] < 0, \tag{27}$$

so both z^* and $\hat{u}^*$ are unique and define a global maximum. Moreover, since z^* is independent of $\hat{u}^*$, optimality can be achieved by first solving for z^* locally and then, given that result, solving for $\hat{u}^*$ locally.[10]

It is interesting to note that no restrictions on $F(a)$ other than continuous differentiability are required to guarantee uniqueness of the first-order conditions for the optimizing firm (who optimally chooses *p* and *k* with the expectation that its demand depends on both). This is in contrast to the literature on the myopic newsvendor (who optimally chooses *p* and *k* with the expectation that its demand depends on *p*, but not on *k*). Demonstrating uniqueness of the optimal policy for the myopic firm typically involves more restrictive conditions on $F(a)$. [11] The simplification arising for the optimizing firm can be attributed to the fact that it chooses the socially efficient z^*,

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[10] In general, if a two-variable function can be written as $f(x,y) = f_1(y)\left[f_2(y) + f_3(y)\right]$, where $f_1 \neq 0$, then the function can be optimized by first treating $f_1(y)$ and $f_2(y)$ as constants to determine x^*, and then treating $f_3(x^*)$ as a constant to determine y^* given x^*.

[11] Petruzzi and Dada (1999) review such conditions. They show, for example, that if $G(\hat{u})$ is such that $G(\hat{u}) = \alpha p^{-\beta}$, then a sufficient condition for uniqueness of the myopic firm's optimal policy is that $F(a)$ satisfy $2h(a) - h'(a) > 0$, where $h(a) = f(a)/[1 - F(a)]$ denotes the hazard rate. It can be shown that this condition is also sufficient for uniqueness when $G(\hat{u}) = \alpha - \beta p$ or $G(\hat{u}) = \alpha e^{-\beta p}$. Note that when $G(\hat{u})$ takes either of these three stated forms our Technical Assumption 5 is satisfied. See also Young (1978).

which is independent of $\hat{u}^*$.[12] It is impossible for the myopic firm to match this efficiency because, to achieve the social optimum, the myopic firm's exogenous price would need to be equal to *V*, but if that were true, then the myopic firm's expected profits would be negative.

Since (20) and (23) are identical, it follows that holding *z* fixed, the myopic firm chooses the same price as the optimizing firm. Intuitively, when the myopic firm considers a price cut it does not internalize the fact that the change in demand might be affected by a change in consumers' expectations of the firm's fill rate. But from (4) a change in $\hat{u}$, holding *z* fixed, has no effect on the fill rate. So, holding *z* fixed, the myopic firm sets the same price as the fully optimizing firm.

However, *z* is not fixed. Comparing (22) to (18), it is clear that $z_m^* < z^*$ since for all $\hat{u} > 0$, $V - \hat{u}/\hat{s} < V$. And by assumption, $R(\hat{u})^2 > R'(\hat{u})$ so $\hat{u} + 1/R(\hat{u})$ is increasing in $\hat{u}$, which, from (20), implies that $\hat{u}_m^* < \hat{u}^*$ as well. To see this it is sufficient to rewrite (20) as

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[12] The social planner maximizes the expected surplus of all consumers less the firm's cost of inventory, however we can ignore consumers who don't value the good since their surplus is independent of *p* and *k*, so the social planner's objective is to maximize

$$V\int_{\underline{a}}^{\bar{a}} \min\left(k, aG(\hat{u})\right) f(a)da + \int_{\underline{a}}^{\bar{a}}\int_{\hat{u}}^{\bar{u}} ug(u)af(a)duda - ck.$$

Using $k = zG(\hat{u})$ and $\mu = E[a]$ we can write this as

$$VG(\hat{u})\left[\mu - \int_{z}^{\bar{a}}(1 - F(a))da\right] + \mu\int_{\hat{u}}^{\bar{u}} ug(u)du - czG(\hat{u}).$$

The derivative with respect to *z* is the same as (22), so the social planner chooses the same stocking factor and service level as the firm. The derivative with respect to $\hat{u}$ is strictly greater than the same derivative in the firm's problem so the social planner chooses a larger outside option for the marginal consumer, sets a lower price, and attracts more consumers. The social planners inventory level is also greater.

$$\frac{\left[V\left[\mu - \int_z^{\bar{a}} [1 - F(a)]da\right] - cz\right]}{\mu} = \hat{u} + \frac{1}{R(\hat{u})}. \tag{28}$$

Since the left-hand side is maximized at z^*, its value must be smaller for any $z < z^*$. Since $z_m^* < z^*$, it follows that $\hat{u}_m^* < \hat{u}^*$. Therefore we can conclude that (i) the myopic stocking factor is less than the optimal stocking factor; (ii) the myopic service level is less than the optimal service level (because $s(z_m^*) < s(z^*)$); and (iii) the myopic inventory level is less than the optimal inventory level (because $k_m^* = z_m^* G(\hat{u}_m^*) < z^* G(\hat{u}^*) = k^*$). However, the relationship between $p_m^* = V - \hat{u}_m^* / s(z_m^*)$ and $p^* = V - \hat{u}^* / s(z^*)$ is ambiguous.

5. Conclusion and Extensions

We have shown that when a firm's demand depends on its inventory policy, then it is optimal to carry more inventory and provide a higher service level than the classic newsvendor model prescribes. The intuition is the same as in models of service rate, or fill rate, competition presented in Carlton (1978) and Deneckere and Peck (1995). These papers assume consumers observe prices and inventories and make a once and for all decision about which firm to visit. However our presentation of a monopoly model when consumers have an exogenous outside option is simpler and suggests a much larger set of environments in which firms' inventory policies should account for the effect of inventory on demand.

The assumptions of our model lead to a demand function that has multiplicative form, but this form is not required for the analysis and its insights to apply. Under different assumptions (e.g., an uncertain number of consumers with no outside option and a predictable number of consumers with an outside option u described by the distribution function $G(u)$) demand would have an additive form. If in addition we assume $\phi(k, \hat{u}) = S(k, \hat{u})$, then we still find that (1) the firm holds more inventory, provides a higher fill rate, and earns higher profits when it internalizes the effect of expected availability on its demand, and (2) in the endogenous price case the firm's two-dimensional decision problem can be reduced to two single variable optimizations that result in the socially efficient stocking factor.[13] However there does not appear to be a reasonable set of assumptions under which demand will take an additive form *and* the consumers' assessment of the probability that they will receive the good if they visit the firm (which, from (5), is the expectation of the ratio of sales and demand) reduces to the firm's fill rate (which, from (3), is the ratio of the expectations of sales and demand). Thus, if demand takes an additive form, then the assumption that $\phi(k,\hat{u}) = S(k,\hat{u})$ does not seem to be justifiable.

The homogeneous valuation assumption can easily be relaxed. Similar results can be shown when consumers have heterogeneous valuations and a homogeneous outside option. If consumers have both heterogeneous outside options and heterogeneous valuations then there will be many marginal consumers. An increase in inventory might attract one consumer with a moderate valuation and moderate outside option as well as another one with a low valuation and a low outside option. Although the analysis would be more complicated, the implications for inventory policy should be very similar.

The assumption that consumers observe the firm's inventory level is clearly very strong. [14] It is sufficient to assume that the consumers observe the firm's fill rate, though arguably this assumption is equally strong. However consumers are likely to use past

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[13] If demand has the form $a + G(\hat{u})$, then the stocking factor is $z = k - G(\hat{u})$ (see Petruzzi and Dada, 1999). With this transformation, the two-variable profit function for the firm who chooses both *p* and *k* can be written in the form $f(x,y) = f_1(y) + f_2(x)$. Thus, the firm's problem reduces to two, independent, single-variable optimization problems.

[14] Dana (2000) looks at an oligopoly model of service rate competition, formally an extension of Deneckere and Peck (1995), in which consumers do not observe firms' inventory levels. He shows that while each firm's optimal inventory policy is the myopic one described by (11) and (19), firms are able to attract more consumers by *raising* their prices. Consumers rationally associate higher service levels with higher priced firms and shop at the firm that maximizes their expected consumer surplus.

experience to estimate the firm's historical fill rate. This model can be interpreted as characterizing the steady state of a repeated game in which firms have reputations for service levels and consumers choose whether to visit the firm based on historical service levels. A firm that maintains a reputation for the optimal service level derived here will be more profitable than one that follows the traditional newsvendor policy. In this case higher stockout rates and lower service levels induce the marginal customer to consume her outside option and the firm is worse off. Credibly raising service levels attracts these customers back to the firm and raises profits. Clearly a better approach would be one in which this dynamic game was modeled explicitly.

The outside option studied here can be more broadly interpreted. Here we have modeled the outside option as an alternative form of consumption, such as watching TV instead of a movie theater. However it may also be the case that it is costly to visit the firm. If consumers have heterogeneous costs u of visiting the firm, then the problem is the same. For example, suppose each consumer who stays home gets 0, and each consumer who visits the firm gets $V - p - u$ if she gets the good or $-u$ if she doesn't. Adding u to each of these payoffs yields the problem studied here. The general point is that when consumers' decision to visit the firm depends on their priors about whether or not the good will be available, then firms' inventory policies should reflect consumer preferences as long as those policies can influence consumers' expectations.

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Appendix A

Lemma: *Equation* (13) *uniquely describes the firm's optimal inventory policy in the exogenous price case.*

Proof: From (9) and (14), the first order condition, (13), can be written as

$$\frac{d\pi(z,\hat{u}(z))}{dz} = \left[p - \frac{c}{[1-F(z)]} + \frac{g(\hat{u}(z))}{G(\hat{u}(z))^2}\pi(z,\hat{u}(z))\frac{(V-p)}{\mu}\right][1-F(z)]G(\hat{u}(z)) = 0. \tag{A1}$$

where $\hat{u}(z)$ is given by (7). From the definition of z_m, it follows that for all $z > z_m$, $p - c/[1-F(z)] < 0$, so (A1) implies

$$\pi(z,\hat{u}(z))\Big|_{\frac{d\pi(z,\hat{u}(z))}{dz}=0} = -\frac{\mu G(\hat{u}(z))^2}{g(\hat{u}(z))(V-p)}\left[p - \frac{c}{1-F(z)}\right] > 0. \tag{A2}$$

Let $b(z)$ denote the term in the first bracket of (A1). Since $[1-F(z)]G(\hat{u}(z)) > 0$, it follows that $sign(d\pi(z,\hat{u}(z))/dz) = sign(b(z))$, and that

$$\begin{aligned} sign\left(\frac{d^2\pi(z,\hat{u}(z))}{dz^2}\Big|_{\frac{d\pi(z,\hat{u}(z))}{dz}=0}\right) &= sign\left(\frac{db(z)}{dz}\Big|_{\frac{d\pi(z,\hat{u}(z))}{dz}=0}\right) \\ &= sign\left(-\frac{cf(z)}{[1-F(z)]^2} + d'(\hat{u}(z))[1-F(z)]\frac{(V-p)^2}{\mu^2}\pi(z,\hat{u}(z))\Big|_{\frac{d\pi(z,\hat{u}(z))}{dz}=0}\right), \end{aligned} \tag{A3}$$

where $d(u)$ denotes $g(u)/G(u)^2$. Since $d(u) = R(u)/G(u)$,

$$d'(u) = \frac{R'(u)G(u) - R(u)g(u)}{G(u)^2} = \frac{\left(R'(u) - R(u)^2\right)}{G(u)} < 0. \tag{A4}$$

Note that $\pi(z,\hat{u}(z))$ is strictly positive when $d\pi(z,\hat{u}(z))/dz = 0$ so it follows that the local second-order condition is satisfied everywhere that the first-order condition is satisfied. Therefore, the first order condition uniquely describes the firm's optimal policy.